CHAPTER 11

Algebra: Linear Functions

510–511 EyeWire

"What does mountain climbing have to do with math?"

As mountain climbers ascend the mountain, the temperature becomes colder. So, the temperature *depends* on the altitude. **In mathematics, you say that the temperature is a function of the altitude.**

You will solve problems about temperature changes and climbing mountains in Lesson 11-3.

GETTING STARTED

Diagnose Readiness

Take this quiz to see if you are ready to begin Chapter 11. Refer to the lesson or page number in parentheses for review.

Vocabulary Review

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. In terms of slope, the rise is the horizontal change. (Lesson 4-3)
2. $y \geq x + 4$ is an example of an inequality. (Lesson 4-3)

Prerequisite Skills

Graph each point on the same coordinate plane. (Page 614)

3. $A(-3, -4)$
4. $B(2, -1)$
5. $C(0, -2)$
6. $D(-4, 3)$

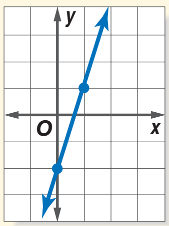
Evaluate each expression if $x = 6$. (Lesson 1-2)

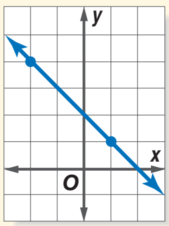
7. $3x$
8. $4x - 9$
9. $2x + 8$
10. $5 + x$

Solve each equation. (Lesson 1-8)

11. $14 = n + 9$
12. $z - 3 = 8$
13. $-17 = b - 21$
14. $23 + r = 16$

Find the slope of each line. (Lesson 4-3)

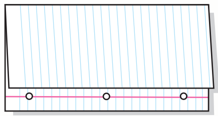
15.


16.


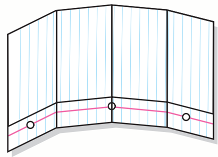
FOLDABLES Study Organizer

Linear Functions Make this Foldable to help you organize your notes. Begin with a plain piece of notebook paper.

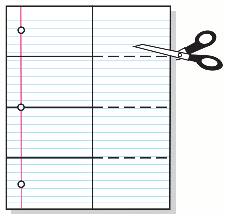
STEP 1 **Fold in Half**
Fold the paper lengthwise to the holes.



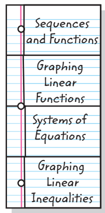
STEP 2 **Fold**
Fold the paper in fourths.



STEP 3 **Cut**
Open. Cut one side along the folds to make four tabs.



STEP 4 **Label**
Label each tab with the main topics as shown.



Chapter Notes Each time you find this logo throughout the chapter, use your *Noteables™: Interactive Study Notebook with Foldables™* or your own notebook to take notes. Begin your chapter notes with this Foldable activity.

Readiness To prepare yourself for this chapter with another quiz, visit **msmath3.net/chapter_readiness**

11-1 Sequences

HANDS-ON Mini Lab

Materials
- toothpicks

What You'll LEARN

Recognize and extend arithmetic and geometric sequences.

NEW Vocabulary

sequence
term
arithmetic sequence
common difference
geometric sequence
common ratio

Work with a partner.

Consider the following pattern.

Number of Triangles	1 triangle	2 triangles	3 triangles
Number of Toothpicks	3 toothpicks	5 toothpicks	7 toothpicks

1. Continue the pattern for 4, 5, and 6 triangles. How many toothpicks are needed for each case?
2. Study the pattern of numbers. How many toothpicks will you need for 7 triangles?

Now, consider another pattern.

Number of Squares	1 square	2 squares	3 squares
Number of Toothpicks	4 toothpicks	7 toothpicks	10 toothpicks

3. Continue the pattern for 4, 5, and 6 squares. How many toothpicks are needed for each case?
4. How many toothpicks will you need for 7 squares?

The numbers of toothpicks needed for each pattern form a sequence. A **sequence** is an ordered list of numbers. Each number is called a **term**.

An **arithmetic sequence** is a sequence in which the difference between any two consecutive terms is the same.

$$3,\ 5,\ 7,\ 9,\ 11,\ \ldots$$

$+2 \quad +2 \quad +2 \quad +2$

The difference is called the **common difference**.

To find the next number in an arithmetic sequence, add the common difference to the last term.

EXAMPLE Identify Arithmetic Sequences

1 **State whether the sequence 17, 12, 7, 2, −3, … is arithmetic. If it is, state the common difference. Write the next three terms of the sequence.**

17, 12, 7, 2, −3 — Notice that $12 - 17 = -5$, $7 - 12 = -5$, and so on.

$-5 \quad -5 \quad -5 \quad -5$

The terms have a common difference of -5, so the sequence is arithmetic. Continue the pattern to find the next three terms.

$$-3, \xrightarrow{-5} -8, \xrightarrow{-5} -13, \xrightarrow{-5} -18$$

The next three terms are -8, -13, and -18.

READING Math

And So On The three dots following a list of numbers are read as *and so on*.

A **geometric sequence** is a sequence in which the quotient between any two consecutive terms is the same.

$$2, \xrightarrow{\times 3} 6, \xrightarrow{\times 3} 18, \xrightarrow{\times 3} 54, \xrightarrow{\times 3} 162, \ldots$$

The quotient is called the **common ratio**.

To find the next number in a geometric sequence, multiply the last term by the common ratio.

EXAMPLES Identify Geometric Sequences

State whether each sequence is geometric. If it is, state the common ratio. Write the next three terms of each sequence.

2 **96, −48, 24, −12, 6, …**

$$96, \xrightarrow{\times\left(-\frac{1}{2}\right)} -48, \xrightarrow{\times\left(-\frac{1}{2}\right)} 24, \xrightarrow{\times\left(-\frac{1}{2}\right)} -12, \xrightarrow{\times\left(-\frac{1}{2}\right)} 6$$

Notice that $-48 \div 96 = -\frac{1}{2}$, $24 \div (-48) = -\frac{1}{2}$, and so on.

The terms have a common ratio of $-\frac{1}{2}$, so the sequence is geometric.

Continue the pattern to find the next three terms.

$$6, \xrightarrow{\times\left(-\frac{1}{2}\right)} -3, \xrightarrow{\times\left(-\frac{1}{2}\right)} \frac{3}{2}, \xrightarrow{\times\left(-\frac{1}{2}\right)} -\frac{3}{4}$$

The next three terms are -3, $\frac{3}{2}$, and $-\frac{3}{4}$.

3 **2, 4, 12, 48, 240, …**

$$2, \xrightarrow{\times 2} 4, \xrightarrow{\times 3} 12, \xrightarrow{\times 4} 48, \xrightarrow{\times 5} 240$$

Since there is no common ratio, the sequence is not geometric. However, the sequence does have a pattern. Multiply the last term by 6, the next term by 7, and the following term by 8.

$$240, \xrightarrow{\times 6} 1{,}440, \xrightarrow{\times 7} 10{,}080, \xrightarrow{\times 8} 80{,}640$$

The next three terms are 1,440, 10,080, and 80,640.

Your Turn **State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio. Write the next three terms of each sequence.**

a. 8, 12, 16, 20, 24, …

b. −5, 25, −125, 625, −3,125, …

c. 1, 2, 4, 7, 11, …

d. 243, 81, 27, 9, 3, …

Skill and Concept Check

1. **Writing Math** Explain how to determine whether a sequence is geometric.

2. **OPEN ENDED** Give a counterexample to the following statement. *All sequences are either arithmetic or geometric.*

3. **Which One Doesn't Belong?** Identify the sequence that is not the same type as the others. Explain your reasoning.

1, 2, 4, 8, 16, ...	125, 25, 5, 1, $\frac{1}{5}$, ...
5, 10, 15, 20, 25, ...	–2, 6, –18, 54, –162, ...

GUIDED PRACTICE

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio. Write the next three terms of each sequence.

4. 2, 4, 6, 8, 10, …

5. 11, 4, -2, -7, -11, …

6. 3, -6, 12, -24, 48, …

FOOD **For Exercises 7–9, use the figure at the right.**

7. Make a list of the number of cans in each level starting with the top.

8. State whether the sequence is *arithmetic*, *geometric*, or *neither*.

9. If the stack of cans had six levels, how many cans would be in the bottom level?

Practice and Applications

HOMEWORK HELP	
For Exercises	See Examples
10–31	1–3

Extra Practice
See pages 642, 658.

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio. Write the next three terms of each sequence.

10. 20, 24, 28, 32, 36, …

11. 1, 10, 100, 1,000, 10,000, …

12. 486, 162, 54, 18, 6, …

13. 88, 85, 82, 79, 76, …

14. 1, 1, 2, 6, 24, …

15. 1, 2, 5, 10, 17, …

16. -6, -4, -2, 0, 2, …

17. 5, -15, 45, -135, 405, …

18. 189, 63, 21, 7, $2\frac{1}{3}$, …

19. 4, $6\frac{1}{2}$, 9, $11\frac{1}{2}$, 14, …

20. 1, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{24}$, $\frac{1}{120}$, …

21. 16, -4, 1, $-\frac{1}{4}$, $\frac{1}{16}$, …

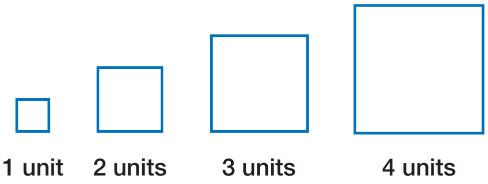
22. -1, 1, -1, 1, -1, …

23. $4\frac{1}{2}$, $4\frac{1}{6}$, $3\frac{5}{6}$, $3\frac{1}{2}$, $3\frac{1}{6}$, …

24. What are the first four terms of an arithmetic sequence with a common difference of $3\frac{1}{3}$ if the first term is 4?

25. What are the first four terms of a geometric sequence with a common ratio of -6 if the first term is 100?

GEOMETRY **For Exercises 26 and 27, use the sequence of squares.**



26. Write a sequence for the areas of the squares. Is the sequence *arithmetic*, *geometric*, or *neither*?

27. Write a sequence for the perimeters of the squares. Is the sequence *arithmetic*, *geometric*, or *neither*?

SKIING **For Exercises 28–32, use the following information.**
A ski resort advertises a one-day lift pass for \$40 and a yearly lift pass for \$400.

28. Copy and complete the table.

Number of Times At the Ski Resort	1	2	3	4	5
Total Cost with One Day Passes	\$40	\$80			
Total Cost with Yearly Pass	\$400	\$400			

29. Is the sequence formed by the row for the total cost with one day passes *arithmetic*, *geometric*, or *neither*?

30. Can the sequence formed by the total cost with a yearly pass be considered arithmetic? Explain.

31. Can the sequence formed by the total cost with a yearly pass be considered geometric? Explain.

32. Extend each sequence to determine how many times a person would have to go skiing to make the yearly pass a better buy.

33. **CRITICAL THINKING** What are the first six terms of an arithmetic sequence where the second term is 5 and the fourth term is 15?

Spiral Review with Standardized Test Practice

34. **MULTIPLE CHOICE** At the beginning of each week, Lina increases the time of her daily jog. If she continues the pattern shown in the table, how many minutes will she spend jogging each day during her fifth week of jogging?

Week	Time Jogging (minutes)
1	8
2	16
3	24
4	32
5	?

(A) 32 min (B) 40 min (C) 48 min (D) 56 min

35. **MULTIPLE CHOICE** Which sequence is geometric?

(F) 5, 5, 10, 30, 120, … (G) −12, −8, −4, 0, 4, …

(H) 1, 1, 2, 3, 5, … (I) −1,280, 320, −80, 20, −5, …

36. **GEOMETRY** The length of a rectangle is 6 inches. Its area is greater than 30 square inches. Write an inequality for the situation. (Lesson 10-7)

Solve each inequality. (Lesson 10-6)

37. $b + 15 > 32$ **38.** $y - 24 \leq 12$ **39.** $9 \leq 16 + t$ **40.** $18 \geq a - 6$

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Evaluate each expression if $x = 9$.** (Lesson 1-2)

41. $2x$ **42.** $x - 12$ **43.** $17 + x$ **44.** $3x - 5$

msmath3.net/self_check_quiz

11-1b HANDS-ON LAB

A Follow-Up of Lesson 11-1

The Fibonacci Sequence

What You'll LEARN

Determine numbers that make up the Fibonacci Sequence.

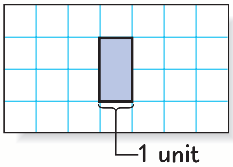
Materials

- grid or dot paper
- colored pencils

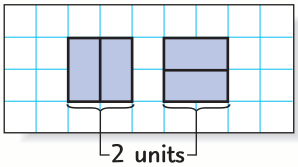
INVESTIGATE *Work in groups of three.*

Leonardo, also known as Fibonacci, created story problems based on a series of numbers that became known as the *Fibonacci sequence*. In this lab, you will investigate this sequence.

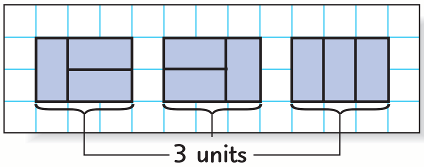
STEP 1 Using grid paper or dot paper, draw a "brick" that is 2 units long and 1 unit wide. If you build a "road" of grid paper bricks, there is only one way to build a road that is 1 unit wide.



STEP 2 Using two bricks, draw all of the different roads that are 2 units wide. There are two ways to build the road.



STEP 3 Using three bricks, draw all of the different roads that are 3 units wide. There are three ways to build the road.



STEP 4 Draw all of the different roads that are 4 units, 5 units, and 6 units long using the brick.

STEP 5 Copy and complete the table.

Length of Road	0	1	2	3	4	5	6
Number of Ways to Build the Road	1	1	2	3			

Writing Math

Work with a partner.

1. **Explain** how each number is related to the previous numbers in the pattern.
2. **Tell** the number of ways there are to build a road that is 8 units long. Do not draw a model.
3. **MAKE A CONJECTURE** Write a rule describing how you generate numbers in the Fibonacci sequence.
4. **RESEARCH** Use the Internet or other resource to find how the Fibonacci sequence is related to nature, music, or art.

11-2 Functions

WHEN am I ever going to use this?

ANIMALS Veterinarians have used the rule that one year of a dog's life is equivalent to seven years of human life.

1. Copy and complete the table at the right.
2. If a dog is 6 years old, what is its equivalent human age?
3. Explain how to find the equivalent human age of a dog that is 10 years old.

Dog's Age	Equivalent Human Age
1	7
2	14
3	
4	
5	

What You'll LEARN

Complete function tables.

NEW Vocabulary

function
function table
independent variable
dependent variable
domain
range

MATH Symbols

$f(x)$ the function of x

The equivalent human age of a dog depends on, or is a function of, its age in years. A relationship where one thing depends upon another is called a **function**. In a function, one or more operations are performed on one number to get another.

Functions are often written as equations.

The *input* x is any real number.

$$f(x) = 7x$$

$f(x)$ is read *the function of* x, or more simply *f of x*. It is the *output*.

The operations performed in the function are sometimes called the *rule*.

To find the value of a function for a certain number, substitute the number into the function value.

EXAMPLES Find a Function Value

Find each function value.

1 $f(9)$ **if** $f(x) = x - 5$

$f(x) = x - 5$

$f(9) = 9 - 5$ or 4 Substitute 9 for x into the function rule.

So, $f(9) = 4$.

2 $f(-3)$ **if** $f(x) = 2x + 1$

$f(x) = 2x + 1$

$f(-3) = 2(-3) + 1$ Substitute -3 for x into the function rule.

$f(-3) = -6 + 1$ or -5 Simplify.

So, $f(-3) = -5$.

Your Turn **Find each function value.**

a. $f(2)$ if $f(x) = x - 4$

b. $f(6)$ if $f(x) = 2x - 8$

READING in the Content Area

For strategies in reading this lesson, visit **msmath3.net/reading**.

Math Online msmath3.net/extra_examples

STUDY TIP

Input and Output The variable for the input is called the **independent variable** because it can be any number. The variable for the output is called the **dependent variable** because it *depends* on the input value.

You can organize the input, rule, and output of a function into a **function table**.

EXAMPLE Make a Function Table

3 **Complete the function table for $f(x) = x + 5$.**

Input	Rule	Output
x	$x + 5$	$f(x)$
-2		
-1		
0		
1		
2		

Substitute each value of x, or input, into the function rule. Then simplify to find the output.

$f(x) = x + 5$

$f(-2) = -2 + 5$ or 3

$f(-1) = -1 + 5$ or 4

$f(0) = 0 + 5$ or 5

$f(1) = 1 + 5$ or 6

$f(2) = 2 + 5$ or 7

Input	Rule	Output
x	$x + 5$	$f(x)$
-2	$-2 + 5$	3
-1	$-1 + 5$	4
0	$0 + 5$	5
1	$1 + 5$	6
2	$2 + 5$	7

The set of input values in a function is called the **domain**. The set of output values is called the **range**. In Example 3, the domain is $\{-2, -1, 0, 1, 2\}$. The range is $\{3, 4, 5, 6, 7\}$.

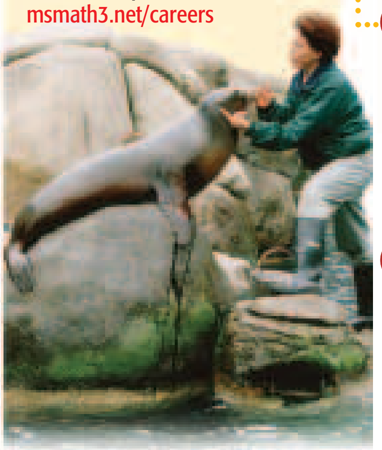
Sometimes functions do not use the $f(x)$ notation. Instead they use two variables. One variable, usually x, represents the input and the other, usually y, represents the output. The function in Example 3 can also be written as $y = x + 5$.

REAL-LIFE CAREERS

How Does a Zookeeper Use Math?

A zookeeper must order the appropriate amount of various foods that will keep their animals healthy.

Math Online Research
For information about a career as a zookeeper, visit msmath3.net/careers



EXAMPLES Functions with Two Variables

ZOOKEEPER **The zoo needs 1.5 tons of specially mixed elephant chow to feed its elephants each week.**

4 **Write a function using two variables to represent the amount of elephant chow needed for w weeks.**

Words Amount of chow equals 1.5 times the number of weeks.

Function $c \quad = \quad 1.5 \quad \cdot \quad w$

The function $c = 1.5w$ represents the situation.

5 **How much elephant chow will the zoo need to feed its elephants for 12 weeks?**

Substitute 12 for w into the function rule.

$c = 1.5w$

$c = 1.5(12)$ or 18 The zoo needs 18 tons of elephant chow.

CONTENTS

Skill and Concept Check

1. **State** the mathematical names for the input values and the output values.

2. **OPEN ENDED** If $f(x) = 2x - 4$, find a value of x that will make the function value a negative number.

3. **FIND THE ERROR** Mitchell and Tomi are finding the function value of $f(x) = 5x$ if the input is 10. Who is correct? Explain.

Mitchell
$f(x) = 5x$
$10 = 5x$
$\frac{10}{5} = \frac{5x}{5}$
$2 = x$

Tomi
$f(x) = 5x$
$f(10) = 5(10)$ or 50

GUIDED PRACTICE

Find each function value.

4. $f(4)$ if $f(x) = x - 6$

5. $f(-2)$ if $f(x) = 4x + 1$

Copy and complete each function table.

6. $f(x) = 8 - x$

x	$8 - x$	$f(x)$
-3		
-1		
2		
4		

7. $f(x) = 5x + 1$

x	$5x + 1$	$f(x)$
-2		
0		
1		
3		

8. $f(x) = 3x - 2$

x	$3x - 2$	y
-5		
-2		
2		
5		

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
9–16	1, 2
17–20	3
21–24	4, 5

Extra Practice
See pages 643, 658.

Find each function value.

9. $f(7)$ if $f(x) = 5x$

10. $f(9)$ if $f(x) = x + 13$

11. $f(4)$ if $f(x) = 3x - 1$

12. $f(5)$ if $f(x) = 2x + 5$

13. $f(-6)$ if $f(x) = -3x + 1$

14. $f(-8)$ if $f(x) = 3x + 24$

15. $f\left(\frac{5}{6}\right)$ if $f(x) = 2x + \frac{1}{3}$

16. $f\left(\frac{5}{8}\right)$ if $f(x) = 4x - \frac{1}{4}$

Copy and complete each function table.

17. $f(x) = 6x - 4$

x	$6x - 4$	$f(x)$
-5		
-1		
2		
7		

18. $f(x) = 5 - 2x$

x	$5 - 2x$	y
-2		
0		
3		
5		

19. $f(x) = 7 + 3x$

x	$7 + 3x$	y
-3		
-2		
1		
6		

20. Make a function table for $y = 3x + 5$ using any four values for x.

GEOMETRY **For Exercises 21 and 22, use the following information.**
The perimeter of a square equals 4 times the length of a side.

21. Write a function using two variables to represent the situation.

22. What is the perimeter of a square with a side 14 inches long?

PARTY PLANNING **For Exercises 23 and 24, use the following information.**
Sherry is having a birthday party at the Swim Center. The cost of renting the pool is \$45 plus \$3.50 for each person.

23. Write a function using two variables to represent the situation.

24. What is the total cost if 20 people attend the party?

25. **WRITE A PROBLEM** Write a real-life problem involving a function.

26. **CRITICAL THINKING** Write the function rule for each function table.

a.

x	$f(x)$
−3	−30
−1	−10
2	20
6	60

b.

x	$f(x)$
−5	−9
−1	−5
3	−1
7	3

c.

x	y
−2	−3
1	3
3	7
5	11

d.

x	y
−2	−5
1	1
3	5
5	9

Spiral Review with Standardized Test Practice

27. **MULTIPLE CHOICE** Which function matches the function table at the right?

x	y
−5	−1
−2	0.2
1	1.4
3	2.2

(A) $y = 0.4x + 1$ (B) $y = 4x - 0.4$

(C) $y = \frac{1}{4}x + 1$ (D) $y = \frac{1}{4}x - 1$

28. **SHORT RESPONSE** A nautical mile is a measure of distance frequently used in sea travel. One nautical mile equals about 6,076 feet. Write a function to represent the number of feet in x nautical miles.

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio. Write the next three terms of each sequence. (Lesson 11-1)

29. 3, −6, 12, −24, 48, … 30. 74, 71, 68, 65, 62, … 31. 2, 3, 5, 8, 12, …

32. **BAND** The school band makes \$0.50 for every candy bar they sell. They want to make at least \$500 on the candy sale. Write and solve an inequality to find how many candy bars they must sell. (Lesson 10-7)

33. **ALGEBRA** Solve $\frac{n}{2} + 31 = 45$. (Lesson 10-2)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Graph each point on the same coordinate plane.** (Page 614)

34. $A(-4, 2)$ 35. $B(3, -1)$ 36. $C(0, -3)$ 37. $D(1, 4)$

11-3a HANDS-ON LAB

A Preview of Lesson 11-3

Graphing Relationships

What You'll LEARN

Graph relationships.

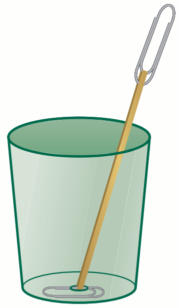
Materials

- pencil
- paper cup
- 2 paper clips
- large rubber band
- tape
- ruler
- 10 pennies
- grid paper

INVESTIGATE *Work in groups of four.*

In this lab, you will investigate a relationship between the number of pennies in a cup and how far the cup will stretch a rubber band.

STEP 1 Using a pencil, punch a small hole in the bottom of the paper cup. Place one paper clip onto the rubber band. Push the other end of the rubber band through the hole in the cup. Attach the second paper clip to the other end of the rubber band. Place it horizontally across the bottom of the cup to keep it from coming through the hole.

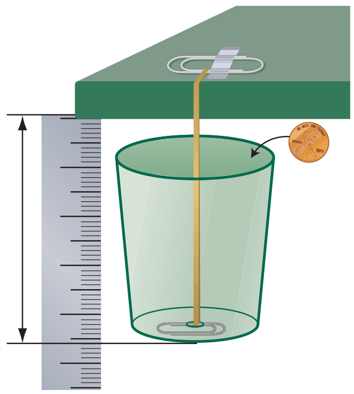


STEP 2 Copy the table at the right.

Number of Pennies x	Distance y

STEP 3 Tape the top paper clip to the edge of a desk. Measure and record the distance from the bottom of the desk to the bottom of the cup. Drop one penny into the cup. Measure and record the new distance from the bottom of the desk to the bottom of the cup.

STEP 4 Continue adding one penny at a time. Measure and record the distance after each addition.



Writing Math

Work with a partner.

1. **Examine** the data. Do you think the number of pennies affects the distance? Explain.
2. **Graph** the ordered pairs formed by your data. Do the points resemble a straight line?
3. **Predict** the distance of the bottom of the cup from the bottom of the desk if 15 pennies are placed in the cup.
4. **Find** the ratio of each distance to the number of pennies. What do you notice about these ratios?

11-3 Graphing Linear Functions

What You'll LEARN

Graph linear functions by using function tables and plotting points.

NEW Vocabulary

linear function
x-intercept
y-intercept

REVIEW Vocabulary

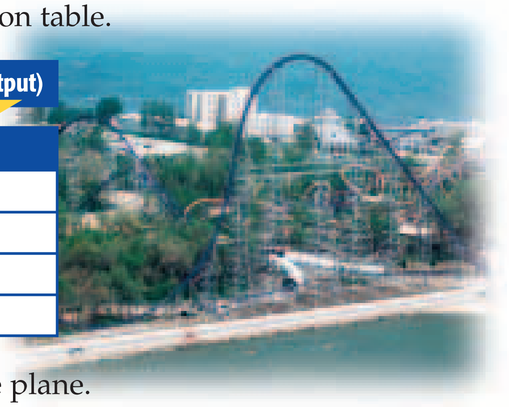
ordered pair: a pair of numbers used to locate a point on a coordinate plane (Lesson 3-6)

WHEN am I ever going to use this?

ROLLER COASTERS The *Millennium Force* roller coaster has a maximum speed of 1.5 miles per minute. If x represents the minutes traveled at this maximum speed, the function rule for the distance traveled is $y = 1.5x$.

1. Copy and complete the following function table.

Input	Rule	Output	(Input, Output)
x	$1.5x$	y	(x, y)
1	1.5(1)	1.5	(1, 1.5)
2	1.5(2)		
3			
4			



2. Graph the ordered pairs on a coordinate plane.
3. What do you notice about the points on your graph?

Ordered pairs of the form (input, output), or (x, y), can represent a function. These ordered pairs can then be graphed on a coordinate plane as part of the graph of the function.

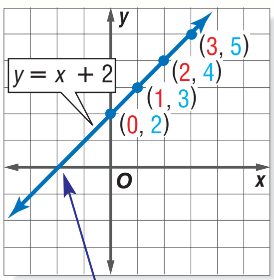
EXAMPLE Graph a Function

1 Graph $y = x + 2$.

Step 1 Choose some values for x. Make a function table. Include a column of ordered pairs of the form (x, y).

x	$x + 2$	y	(x, y)
0	0 + 2	2	(0, 2)
1	1 + 2	3	(1, 3)
2	2 + 2	4	(2, 4)
3	3 + 2	5	(3, 5)

Step 2 Graph each ordered pair. Draw a line that passes through each point. Note that the ordered pair for any point on this line is a solution of $y = x + 2$. The line is the complete graph of the function.



The point where the line crosses the x-axis is the solution to the equation $0 = x + 2$.

Check It appears from the graph that $(-2, 0)$ is also a solution. Check this by substitution.

$y = x + 2$ Write the function.

$0 \stackrel{?}{=} -2 + 2$ Replace x with -2 and y with 0.

$0 = 0$ ✔ Simplify.

A function in which the graph of the solutions forms a line is called a **linear function**. Therefore, $y = x + 2$ is a *linear equation.*

Concept Summary — **Representing Functions**

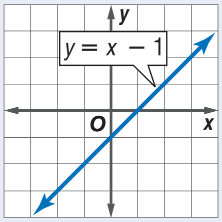
Words The value of y is one less than the corresponding value of x.

Equation $y = x - 1$ **Ordered Pairs** $(0, -1), (1, 0), (2, 1), (3, 2)$

Table

x	y
0	−1
1	0
2	1
3	2

Graph

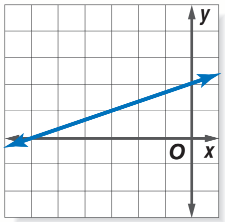


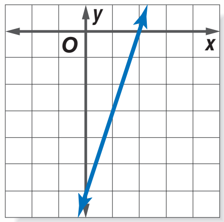
The value of x where the graph crosses the x-axis is called the **x-intercept**. The value of y where the graph crosses the y-axis is called the **y-intercept**.

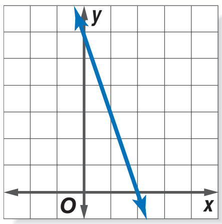
EXAMPLE Use x- and y-intercepts

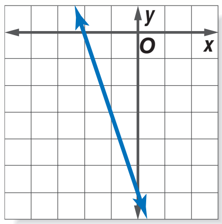
Standardized Test Practice

2 **MULTIPLE-CHOICE TEST ITEM** Which graph represents $y = 3x - 6$?

A


B


C


D


Test-Taking Tip

Use Different Methods Always work each problem using the method that is easiest for you. You could solve the problem at the right in several ways.

- You could test the coordinates of several points on each graph.
- You could graph the function and see which graph matched your graph.
- You could determine the intercepts and see which graph had those intercepts.

Which method is easiest for you?

Read the Test Item You need to decide which of the four graphs represents $y = 3x - 6$.

Solve the Test Item

The graph will cross the x-axis when $y = 0$.

$0 = 3x - 6$	Let $y = 0$.
$0 + 6 = 3x - 6 + 6$	Add 6.
$6 = 3x$	Simplify.
$\frac{6}{3} = \frac{3x}{3}$	Divide by 3.
$2 = x$	Simplify.

The graph will cross the y-axis when $x = 0$.

$y = 3(0) - 6$	Let $x = 0$.
$y = 0 - 6$	Simplify.
$y = -6$	Simplify.

The x-intercept is 2, and the y-intercept is -6. Graph B is the only graph with both of these intercepts. The answer is B.

Skill and Concept Check

1. **Writing Math** Explain how a function table can be used to graph a function.

2. **OPEN ENDED** Draw a graph of a linear function. Name the coordinates of three points on the graph.

3. **Which One Doesn't Belong?** Identify the ordered pair that is not a solution of $y = 2x - 3$. Explain your reasoning.

(1, –1)	(2, 1)	(0, 3)	(–2, –7)

GUIDED PRACTICE

4. Copy and complete the function table at the right. Then graph $y = x + 5$.

x	$x + 5$	y	(x, y)
−3			
−1			
0			
2			

Graph each function.

5. $y = 3x$
6. $y = 3x + 1$
7. $y = \frac{x}{2} - 1$

FOOD **For Exercises 8 and 9, use the following information.**
The function $y = 40x$ describes the relationship between the number of gallons of sap y used to make x gallons of maple syrup.

8. Graph the function.
9. From your graph, how much sap is needed to make to make $2\frac{1}{2}$ gallons of syrup?

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
10–23	1
27	2

Extra Practice
See pages 643, 658.

Copy and complete each function table. Then graph the function.

10. $y = x - 4$

x	$x - 4$	y	(x, y)
−1			
1			
3			
5			

11. $y = 2x$

x	$2x$	y	(x, y)
−2			
0			
1			
2			

Graph each function.

12. $y = 4x$
13. $y = -3x$
14. $y = x - 3$
15. $y = x + 1$
16. $y = 3x - 7$
17. $y = 2x + 3$
18. $y = \frac{x}{3} + 1$
19. $y = \frac{x}{2} - 3$

20. Draw the graph of $y = 5 - x$.
21. Graph the function $y = -\frac{1}{2}x + 5$.

22. **GEOMETRY** The equation $s = 180(n - 2)$ relates the sum of the measures of angles s formed by the sides of a polygon to the number of sides n. Find four ordered pairs (n, s) that are solutions of the equation.

MOUNTAIN CLIMBING For Exercises 23 and 24, use the following information.

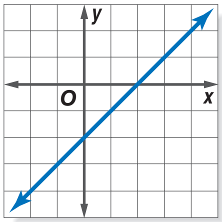
If the temperature is 80°F at sea level, the function $t = 80 - 3.6h$ describes the temperature t at a height of h thousand feet above sea level.

23. Graph the temperature function.

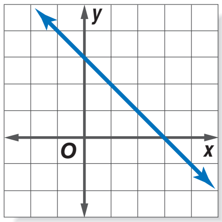
24. The top of Mount Everest is about 29 thousand feet above sea level. What is the temperature at its peak on a day that is 80°F at sea level?

25. **CRITICAL THINKING** Name the coordinates of four points that satisfy each function. Then give the function rule.

a.



b.



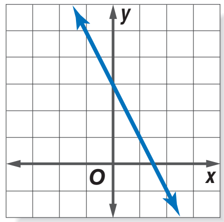
26. **CRITICAL THINKING** The vertices of a triangle are at $(-1, -1)$, $(1, -2)$, and $(5, 1)$. The triangle is translated 1 unit left and 2 units up and then reflected across the graph of $y = x - 1$. What are the coordinates of the image? (*Hint:* Use a ruler.)

Spiral Review with Standardized Test Practice

27. **MULTIPLE CHOICE** Which function is graphed at the right?

Ⓐ $y = -2x + 3$　　Ⓑ $y = -2x - 3$

Ⓒ $y = 2x + 3$　　Ⓓ $y = 2x - 3$



28. **SHORT RESPONSE** An African elephant eats 500 pounds of vegetation a day. Write a function for the amount of vegetation y it eats in x days. Graph the function.

Find each function value. (Lesson 11-2)

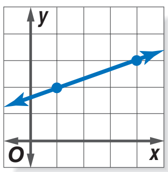
29. $f(6)$ if $f(x) = 7x - 3$　　30. $f(-5)$ if $f(x) = 3x + 15$　　31. $f(3)$ if $f(x) = 2x - 7$

32. **SCIENCE** Each time a certain ball hits the ground, it bounces up $\frac{2}{3}$ of its previous height. If the ball is dropped from 27 inches off the ground, write a sequence showing the height of the ball after each of the first three bounces. (Lesson 11-1)

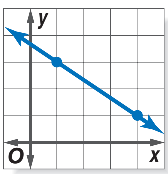
GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Find the slope of each line. (Lesson 4-3)

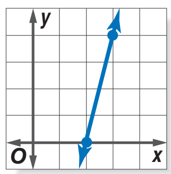
33.



34.



35.



11-4 The Slope Formula

What You'll LEARN

Find the slope of a line using the slope formula.

NEW Vocabulary

slope formula

REVIEW Vocabulary

slope: the ratio of the rise, or vertical change, to the run, or horizontal change (Lesson 4-3)

MATH Symbols

x_2 x sub 2

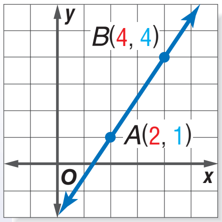
HANDS-ON Mini Lab

Materials

- grid paper

Work with a partner.

On a coordinate plane, graph $A(2, 1)$ and $B(4, 4)$. Draw the line through points A and B as shown.



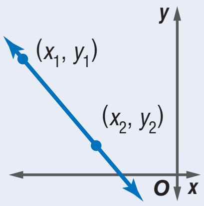
1. Find the slope of the line by counting units of vertical and horizontal change.
2. Subtract the y-coordinate of A from the y-coordinate of B. Call this value t.
3. Subtract the x-coordinate of A from the x-coordinate of B. Call this value s.
4. Write the ratio $\frac{t}{s}$. Compare the slope of the line with $\frac{t}{s}$.

You can find the slope of a line by using the coordinates of any two points on the line. One point can be represented by (x_1, y_1) and the other by (x_2, y_2). The small numbers slightly below x and y are called *subscripts*.

Noteables™ **Key Concept: Slope Formula**

Words The slope m of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y-coordinates to the corresponding difference in the x-coordinates.

Model



Symbols $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 \neq x_1$

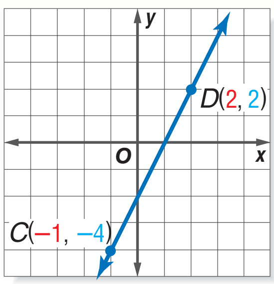
EXAMPLE Positive Slope

1 **Find the slope of the line that passes through $C(-1, -4)$ and $D(2, 2)$.**

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{2 - (-4)}{2 - (-1)}$ $(x_1, y_1) = (-1, -4)$, $(x_2, y_2) = (2, 2)$

$m = \frac{6}{3}$ or 2 Simplify.



Check When going from left to right, the graph of the line slants upward. This is consistent with a positive slope.

STUDY TIP

Using the Slope Formula

- It does not matter which point you define as (x_1, y_1) and (x_2, y_2).
- However, the coordinates of both points must be used in the same order.

Check In Example 2, let $(x_1, y_1) = (-4, 3)$ and $(x_2, y_2) = (1, 2)$. Then find the slope.

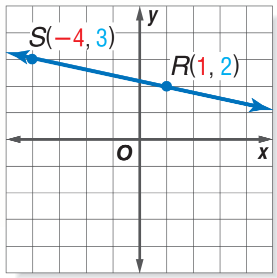
EXAMPLE Negative Slope

2 **Find the slope of the line that passes through $R(1, 2)$ and $S(-4, 3)$.**

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \dfrac{3 - 2}{-4 - 1}$ $(x_1, y_1) = (1, 2)$, $(x_2, y_2) = (-4, 3)$

$m = \dfrac{1}{-5}$ or $-\dfrac{1}{5}$ Simplify.



Check When going from left to right, the graph of the line slants downward. This is consistent with a negative slope.

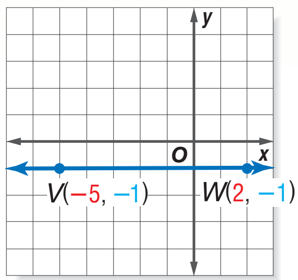
EXAMPLE Zero Slope

3 **Find the slope of the line that passes through $V(-5, -1)$ and $W(2, -1)$.**

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \dfrac{-1 - (-1)}{2 - (-5)}$ $(x_1, y_1) = (-5, -1)$, $(x_2, y_2) = (2, -1)$

$m = \dfrac{0}{7}$ or 0 Simplify.



The slope is 0. The slope of any horizontal line is 0.

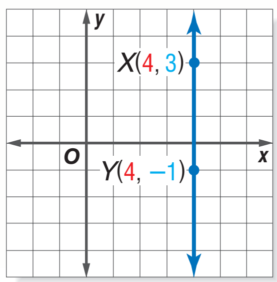
EXAMPLE Undefined Slope

4 **Find the slope of the line that passes through $X(4, 3)$ and $Y(4, -1)$**

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \dfrac{-1 - 3}{4 - 4}$ $(x_1, y_1) = (4, 3)$, $(x_2, y_2) = (4, -1)$

$m = \cancel{\dfrac{-4}{0}}$ Simplify.



Division by 0 is not defined. So, the slope is undefined. The slope of any vertical line is undefined.

Your Turn **Find the slope of the line that passes through each pair of points.**

a. $M(2, 2)$, $N(5, 3)$

b. $A(-2, 1)$, $B(0, -3)$

c. $C(-5, 6)$, $D(-5, 0)$

d. $E(-1, 1)$, $F(3, 1)$

Skill and Concept Check

1. **Writing Math** Explain why the slope formula, which states $m = \frac{y_2 - y_1}{x_2 - x_1}$, says that x_2 cannot equal x_1.

2. **OPEN ENDED** Write the coordinates of two points. Show that you can define either point as (x_1, y_1) and the slope of the line containing the points will be the same.

3. **FIND THE ERROR** Martin and Dylan are finding the slope of the line that passes through $X(0, 2)$ and $Y(2, 3)$. Who is correct? Explain.

Martin

$m = \frac{3-2}{0-2}$

$m = \frac{1}{-2}$ or $-\frac{1}{2}$

Dylan

$m = \frac{3-2}{2-0}$

$m = \frac{1}{2}$

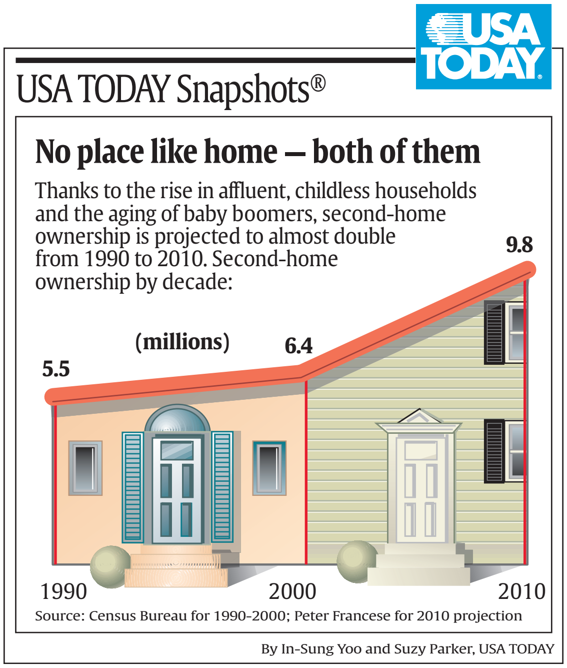
GUIDED PRACTICE

Find the slope of the line that passes through each pair of points.

4. $A(-3, -2)$, $B(5, 4)$
5. $C(-4, 2)$, $D(1, 2)$
6. $E(-6, 5)$, $F(3, -3)$

For Exercises 7–9, use the graphic at the right. Note that the years are on the horizontal axis and the second homeownership is on the vertical axis.

7. Find the slope of the line representing the change from 1990 to 2000.
8. Find the slope of the line representing the change from 2000 to 2010.
9. Which part of the graph shows a greater rate of change? Explain.



Practice and Applications

Find the slope of the line that passes through each pair of points.

10. $A(0, 1)$, $B(2, 7)$
11. $C(2, 5)$, $D(3, 1)$
12. $E(1, 2)$, $F(4, 7)$
13. $G(-6, -1)$, $H(4, 1)$
14. $J(-9, 3)$, $K(2, 1)$
15. $M(-2, 3)$, $N(7, -4)$
16. $P(4, -4)$, $Q(8, -4)$
17. $R(-1, 5)$, $S(-1, -2)$
18. $T(3, -2)$, $U(3, 2)$
19. $V(-6, 5)$, $W(3, -3)$
20. $X(21, 5)$, $Y(17, 0)$
21. $Z(24, 12)$, $A(34, 2)$

HOMEWORK HELP

For Exercises	See Examples
10–21	1–4

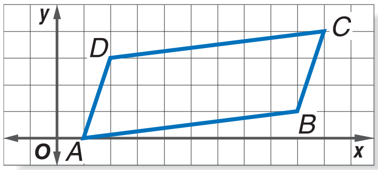
Extra Practice
See pages 643, 658.

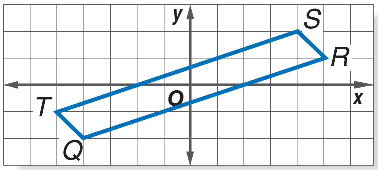
TRAVEL **For Exercises 22 and 23, use the following information.**
After 2 hours, Kendra has traveled 110 miles. After 3 hours, she has traveled 165 miles. After 5 hours, she has traveled 275 miles.

22. Graph the information with the hours on the horizontal axis and miles traveled on the vertical axis. Draw a line through the points.

23. What is the slope of the graph? What does it represent?

GEOMETRY **For Exercises 24 and 25, use the following information to show that each quadrilateral graphed is a parallelogram.**
Two lines that are parallel have the same slope.

24.


25.


26. **CRITICAL THINKING** Without graphing, determine whether $A(5, 1)$, $B(1, 0)$, and $C(-3, -3)$ lie on the same line. Explain.

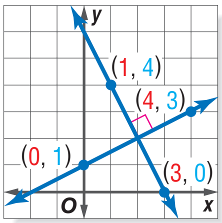
EXTENDING THE LESSON

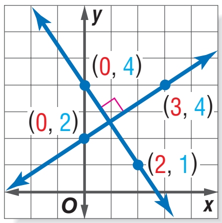
For Exercises 27–29, use the graphs at the right. The two lines in each graph are perpendicular.

27. Find the slopes of the lines in graph A.

28. Find the slopes of the lines in graph B.

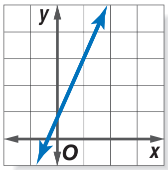
29. **Make a conjecture** about the slopes of perpendicular lines.

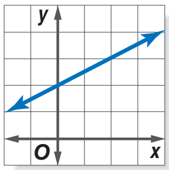
Graph A


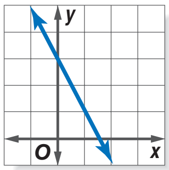
Graph B


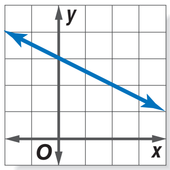
Spiral Review with Standardized Test Practice

30. **MULTIPLE CHOICE** Which graph has a slope of -2?

(A)


(B)


(C)


(D)


31. **SHORT RESPONSE** Draw a graph of a line with an undefined slope.

Graph each function. (Lesson 11-3)

32. $y = 5x$
33. $y = x - 2$
34. $y = 2x - 1$
35. $y = 3x + 2$

36. **TEMPERATURE** The function used to change a Celsius temperature (C) to a Fahrenheit temperature (F) is $F = \frac{9}{5}C + 32$. Change 25° Celsius to degrees Fahrenheit. (Lesson 11-2)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Solve each equation.** (Lesson 1-8)

37. $7 + a = 15$
38. $23 = d + 44$
39. $28 = n - 14$
40. $t - 22 = -31$

CHAPTER 11

Mid-Chapter Practice Test

Vocabulary and Concepts

1. **Explain** how to find the next three terms of the sequence 5, 8, 11, 14, 17, … . (Lesson 11-1)

2. **Explain** how to graph $y = -2x + 1$. (Lesson 11-3)

3. **Describe** the slopes of a horizontal line and a vertical line. (Lesson 11-4)

Skills and Applications

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio. Then write the next three terms of the sequence. (Lesson 11-1)

4. 13, 17, 21, 25, 29, …
5. 64, −32, 16, −8, 4, …
6. 5, 6, 8, 11, 15, …

7. **PICNICS** Shelby is hosting a picnic. The cost to rent the shelter is $25 plus $2 per person. Write a function using two variables to represent the situation. Find the total cost if 150 people attend. (Lesson 11-2)

Graph each function. (Lesson 11-3)

8. $y = -2x$
9. $y = x + 6$
10. $y = 2x - 5$

Find the slope of the line that passes through each pair of points. (Lesson 11-4)

11. $A(2, 5)$, $B(3, 1)$
12. $C(-1, 2)$, $D(-5, 2)$
13. $E(5, 2)$, $F(2, -3)$

Standardized Test Practice

14. **MULTIPLE CHOICE** Which equation describes the function represented by the table? (Lesson 11-2)

x	$f(x)$
−2	−7
0	−3
2	1
4	5

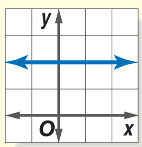
Ⓐ $f(x) = 2x - 3$

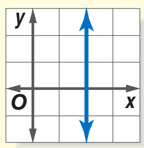
Ⓑ $f(x) = x + 4$

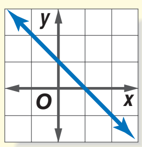
Ⓒ $f(x) = n - 3$

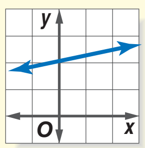
Ⓓ $f(x) = 2x + 3$

15. **MULTIPLE CHOICE** Which graph has a negative slope? (Lesson 11-4)

Ⓕ 

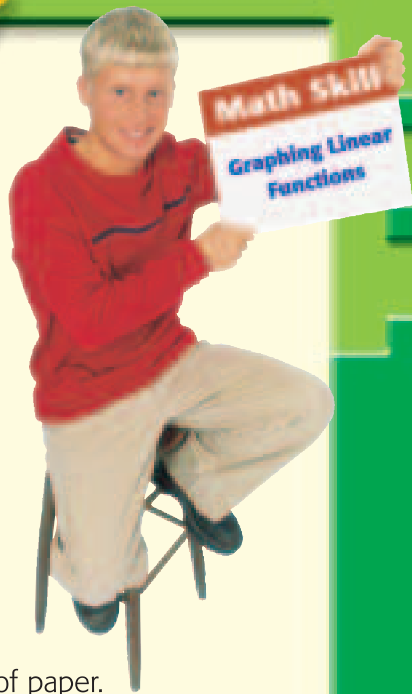
Ⓖ 

Ⓗ 

Ⓘ 

The Game Zone

A Place To Practice Your Math Skills



It's a Hit

GET READY!

Players: two
Materials: large piece of paper, marker, grid paper

GET SET!

- Use a marker to list the following functions on a piece of paper.

$y = -2x$	$y = x + 2$	$y = x - 2$
$y = -x + 2$	$y = 3 - x$	$y = 1 - 2x$
$y = 2x$	$y = -x - 1$	$y = x - 1$
$y = x + 1$	$y = 2x - 1$	$y = -x - 2$
$y = x - 3$	$y = 2x + 1$	$y = -x + 1$

- Each player makes two coordinate planes. Each plane should be on a 20-by-20 grid with the origin in the center.

GO!

- Each player secretly picks one of the functions listed on the paper and graphs it on one of his or her coordinate planes.
- The first player names an ordered pair. The second player says *hit* if the ordered pair names a point on his or her line. If not, the player says *miss*.
- Then the second player names an ordered pair. It is either a *hit* or a *miss*. Players should use their second coordinate plane to keep track of their hits and misses. Players continue to take turns guessing.
- **Who Wins?** A player who correctly names the equation of the other player is the winner. However, if a player incorrectly names the equation, the other player is the winner.

11-5a

Graphing Calculator Investigation

A Preview of Lesson 11-5

What You'll LEARN

Use a graphing calculator to graph families of lines.

Families of Linear Graphs

Families of graphs are graphs that are related in some manner. In this investigation, you will study families of linear graphs.

ACTIVITY

Graph $y = -2x + 4$, $y = -2x + 1$, and $y = -2x - 3$.

STEP 1 **Clear any existing equations from the Y= list.**

Keystrokes: [Y=] [CLEAR]

STEP 2 **Enter each equation.**

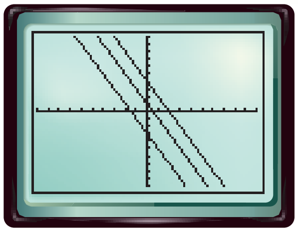
Keystrokes: [(–)] 2 [X,T,θ,n] [+] 4 [ENTER]

[(–)] 2 [X,T,θ,n] [+] 1 [ENTER]

[(–)] 2 [X,T,θ,n] [–] 3 [ENTER]

STEP 3 **Graph the equations in the standard viewing window.**

Keystrokes: [ZOOM] 6



EXERCISES

1. **Compare** the three equations.
2. **Describe** the graphs of the three equations.
3. **MAKE A CONJECTURE** Consider equations of the form $y = ax + b$, where the value of a is the same but the value of b varies. What do you think is true about the graphs of the equations?
4. **Use a graphing calculator** to graph $y = 2x + 3$, $y = -x + 3$, and $y = -3x + 3$.
5. **Compare** the three equations you graphed in Exercise 4.
6. **Describe** the graphs of the three equations you graphed in Exercise 4.
7. **MAKE A CONJECTURE** Consider equations of the form $y = ax + b$, where the value of a changes but the value of b remains the same. What do you think is true about the graphs of the equations?
8. **Write** equations of three lines whose graphs are a family of graphs. Describe the common characteristic of the graph.

Math Online msmath3.net/other_calculator_keystrokes

11-5 Slope-Intercept Form

HANDS-ON Mini Lab

Materials

- grid paper

Work with a partner.

Graph each equation listed in the table at the right.

Equation	Slope	y-intercept
$y = 3x + 2$		
$y = \frac{1}{4}x + (-1)$		
$y = -2x + 3$		

1. Use the graphs to find the slope and y-intercept of each line. Copy and complete the table.
2. Compare each equation with the value of its slope. What do you notice?
3. Compare each equation with its y-intercept. What do you notice?

What You'll LEARN

Graph linear equations using the slope and y-intercept.

NEW Vocabulary

slope-intercept form

Link to READING

Everyday Meaning of Intercept: to intersect or cross

All of the equations in the table above are written in the form $y = mx + b$. This is called the **slope-intercept form**. When an equation is written in this form, m is the slope, and b is the y-intercept.

$$y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{y\text{-intercept}}$$

EXAMPLES Find Slopes and y-intercepts of Graphs

State the slope and the y-intercept of the graph of each equation.

1 $y = \frac{2}{3}x - 4$

$y = \frac{2}{3}x + (-4)$ Write the equation in the form $y = mx + b$.

$\uparrow \quad \uparrow$

$y = mx + b$ $m = \frac{2}{3}, b = -4$

The slope of the graph is $\frac{2}{3}$, and the y-intercept is -4.

2 $x + y = 6$

$x + y = 6$ Write the original equation.

$-x \qquad -x$ Subtract x from each side.

$y = 6 - x$ Simplify.

$y = -1x + 6$ Write the equation in the form $y = mx + b$. Recall that $-x$ means $-1x$.

$\uparrow \quad \uparrow$

$y = mx + b$ $m = -1, b = 6$

The slope of the graph is -1, and the y-intercept is 6.

You can use the slope-intercept form of an equation to graph the equation.

EXAMPLE Graph an Equation

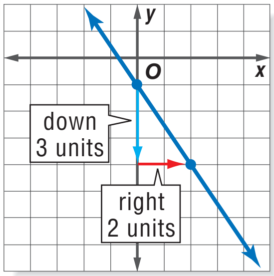
3 **Graph $y = -\frac{3}{2}x - 1$ using the slope and y-intercept.**

Step 1 Find the slope and y-intercept.

$$y = -\frac{3}{2}x - 1$$

slope $= -\frac{3}{2}$ y-intercept $= -1$

Step 2 Graph the y-intercept $(0, -1)$.



Step 3 Write the slope $-\frac{3}{2}$ as $\frac{-3}{2}$. Use it to locate a second point on the line.

$m = \frac{-3}{2}$ ← change in y: down 3 units
← change in x: right 2 units

Step 4 Draw a line through the two points.

Your Turn **Graph each equation using the slope and y-intercept.**

a. $y = x + 3$ b. $y = \frac{1}{2}x - 1$ c. $y = -\frac{4}{3}x + 2$

REAL-LIFE MATH

ADVERTISING In the year 2000, over $236 billion was spent on advertising in the United States.

Source: McCann-Erickson, Inc.

EXAMPLES Graph an Equation to Solve Problems

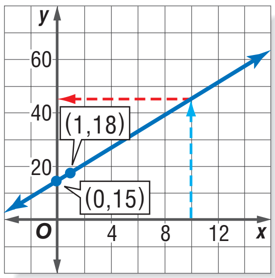
ADVERTISING **Student Council wants to buy posters advertising the school's carnival. The Design Shoppe charges $15 to prepare the design and $3 for each poster printed. The total cost y can be represented by the equation $y = 3x + 15$, where x represents the number of posters.**

4 **Graph the equation.**

First find the slope and the y-intercept.

$$y = 3x + 15$$

slope y-intercept



Plot the point (0, 15). Then locate another point up 3 and right 1. Draw the line.

5 **Use the graph to find the cost for 10 posters.**

Locate 10 on the x-axis. Find the y-coordinate on the graph where the x-coordinate is 10. The total cost is $45.

6 **Describe what the slope and y-intercept represent.**

The slope 3 represents the cost per poster, which is the rate of change. The y-intercept 15 is the one-time charge for preparing the design.

Juan Silva/Getty Images

Skill and Concept Check

1. **Writing Math** Explain how to graph a line with a slope of $-\frac{5}{4}$ and a y-intercept of -3.

2. **OPEN ENDED** Draw the graph of a line that has a y-intercept but no x-intercept. What is the slope of the line?

3. **Which One Doesn't Belong?** Identify the equation that has a graph with a different slope. Explain your reasoning.

$y = \frac{2}{3}x - 4$	$y = \frac{3}{2}x + 1$	$y = \frac{2}{3}x + 7$	$y = \frac{2}{3}x$

GUIDED PRACTICE

State the slope and the y-intercept for the graph of each equation.

4. $y = x + 2$
5. $y = -\frac{1}{6}x - \frac{1}{2}$
6. $2x + y = 3$

Graph each equation using the slope and the y-intercept.

7. $y = \frac{1}{3}x - 2$
8. $y = -\frac{5}{2}x + 1$
9. $y = -2x + 5$

MONEY MATTERS For Exercises 10–12, use the following information.
Lydia borrowed \$90 and plans to pay back \$10 per week. The equation for the amount of money y Lydia owes after x weeks is $y = 90 - 10x$.

10. Graph the equation.
11. What does the slope of the graph represent?
12. What does the x-intercept of the graph represent?

Practice and Applications

State the slope and the y-intercept for the graph of each equation.

13. $y = 3x + 4$
14. $y = -5x + 2$
15. $y = \frac{1}{2}x - 6$
16. $y = -\frac{3}{7}x - \frac{1}{7}$
17. $y - 2x = 8$
18. $3x + y = -4$

19. Graph a line with a slope of $\frac{1}{2}$ and a y-intercept of -3.

20. Graph a line with a slope of $-\frac{2}{3}$ and a y-intercept of 0.

21. Write an equation in slope-intercept form of the line with a slope of -2 and a y-intercept of 6.

22. Write an equation in slope-intercept form of the line with slope of 4 and a y-intercept of -10.

HOMEWORK HELP

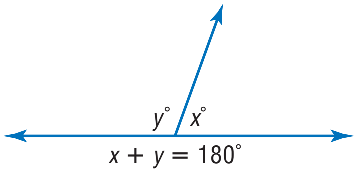
For Exercises	See Examples
13–18, 21–22	1, 2
19–20, 23–30	3
31–36	4–6

Extra Practice
See pages 644, 658.

Graph each equation using the slope and the y-intercept.

23. $y = \frac{1}{3}x - 5$
24. $y = -x + \frac{3}{2}$
25. $y = -\frac{4}{3}x + 1$
26. $y = \frac{3}{2}x - 4$
27. $y = -2x - 3.5$
28. $y = 3x + 1.5$
29. $y - 3x = 5$
30. $5x + y = -2$

GEOMETRY **For Exercises 31–33, use the information at the right.**



31. Write the equation in slope-intercept form.

32. Graph the equation.

33. Use the graph to find the value of y if $x = 70$.

SPACE SCIENCE **For Exercises 34–36, use the following information.** From 4,074 meters above Earth, the space shuttle Orbiter glides to the runway. Let $y = 4{,}074 - 47x$ represent the altitude of the Orbiter after x seconds.

34. Graph the equation.

35. What does the slope of the graph represent?

36. What does the x-intercept of the graph represent?

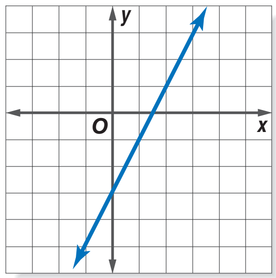
37. **WRITE A PROBLEM** Write a real-life problem that involves a linear equation in slope-intercept form. Graph the equation. Explain the meaning of the slope and y-intercept.

38. Is it *sometimes*, *always*, or *never* possible to draw more than one line given a slope and a y-intercept? Explain.

39. **CRITICAL THINKING** Suppose the graph of a line is vertical. What is the slope and y-intercept of the line?

Spiral Review with Standardized Test Practice

40. **MULTIPLE CHOICE** What is the equation of the graph at the right?



Ⓐ $y = \frac{1}{2}x - 3$ Ⓑ $y = -\frac{1}{2}x - 3$

Ⓒ $y = 2x - 3$ Ⓓ $y = -2x - 3$

41. **MULTIPLE CHOICE** A taxi fare y can be determined by the equation $y = 3x + 5$, where x is the number of miles traveled. What does the slope of the graph of this equation represent?

Ⓕ the distance traveled Ⓖ the cost per mile

Ⓗ the initial fare Ⓘ none of the above

Find the slope of the line that passes through each pair of points. (Lesson 11-4)

42. $M(4, 3)$, $N(-2, 1)$ **43.** $S(-5, 4)$, $T(-7, 1)$ **44.** $X(-9, 5)$, $Y(-2, 5)$

45. **MEASUREMENT** The function $y = 0.39x$ approximates the number of inches y in x centimeters. Make a function table. Then graph the function. (Lesson 11-3)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Graph each point on the same coordinate plane.** (Page 614)

46. $A(5, 2)$ **47.** $B(1.5, 2.5)$ **48.** $C(2.3, 1.8)$ **49.** $D(7.5, 3.2)$

11-6a Problem-Solving Strategy

A Preview of Lesson 11-6

Use a Graph

What You'll LEARN

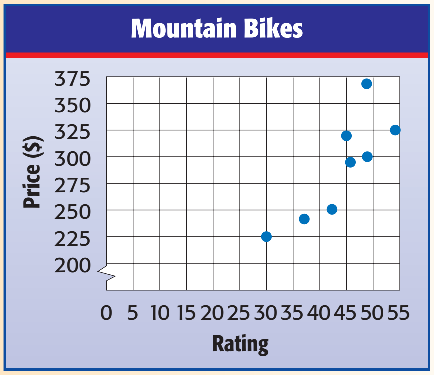
Solve problems by using graphs.

I want to buy a mountain bike. I made a graph with the ratings and the prices of 8 different bikes.

Are the highest rated bikes the most expensive bikes?

Explore

We have a graph. We want to know whether the highest rated bikes are the most expensive.



Higher ratings represent better bikes.

Plan

Let's study the graph.

Solve

The graph shows that the highest rated bike is *not* the most expensive bike. Also the prices of the two bikes with the second highest rating vary considerably.

Examine

Look at the graph. The dot farthest to the right is *not* the highest on the graph.

Analyze the Strategy

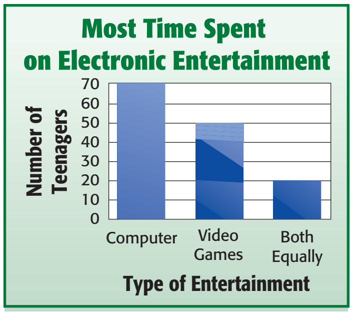
1. **Explain** why the bike represented by (48, 300) might be the best bike to buy.
2. **Find** a graph in a newspaper, magazine, or the Internet. Write a sentence explaining the information contained in the graph.

Laura Sifferlin

Apply the Strategy

Solve. Use a graph.

3. **TECHNOLOGY** Teenagers were asked which they spent more time using their computer, their video game system, or both equally. The graph shows the results of the survey. How many teenagers were surveyed?



4. **ZOOLOGY** A zoologist studied extinction times in years of birds on an island. Make a graph of the data. Does the bird with the greatest average number of nests have the greatest extinction time?

Bird	Average Number of Nests	Extinction Time (yr)
Cuckoo	1.4	2.5
Magpie	4.5	10.0
Swallow	3.8	2.6
Robin	3.3	4.0
Stonechat	3.6	2.4
Blackbird	4.7	3.3
Tree-Sparrow	2.2	1.9

Mixed Problem Solving

Solve. Use any strategy.

5. **MULTI STEP** Caton's big brother has a full scholarship for tuition, books, and room and board for four years of college. The total scholarship is $87,500. Room and board cost $9,500 per year. His books cost about $750 per year. What is the cost of his yearly tuition?

EDUCATION **For Exercises 6 and 7, use the table below.**

Students per Computer in U.S. Public Schools			
Year	**Students**	**Year**	**Students**
1991	20	1996	10
1992	18	1997	7.8
1993	16	1998	6.1
1994	14	1999	5.7
1995	10.5	2000	5.4

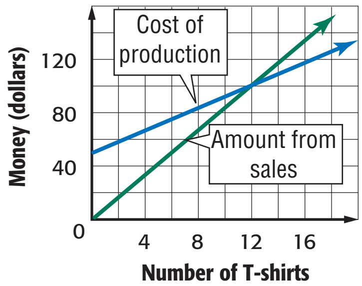
Source: National Center for Education Statistics

6. Make a graph of the data.

7. Describe how the number of students per computer changed from 1991 to 2000.

8. **MONEY MATTERS** Francisco spent twice as much on athletic shoes as he did on a new pair of jeans. The total bill came to $120. What was the cost of his new jeans?

9. **STANDARDIZED TEST PRACTICE** The blue line shows the cost of producing T-shirts. The green line shows the amount of money received from the sales of the T-shirts. How many shirts must be sold to make a profit?



Ⓐ less than 12 T-shirts

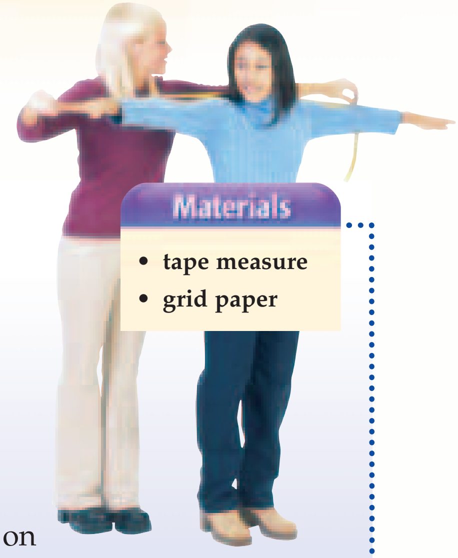
Ⓑ exactly 12 T-shirts

Ⓒ more than 12 T-shirts

Ⓓ cannot be determined from the graph

11-6 Statistics: Scatter Plots

HANDS-ON Mini Lab



What You'll LEARN

Construct and interpret scatter plots.

NEW Vocabulary

scatter plot
best-fit line

Work with a partner.

Materials
- tape measure
- grid paper

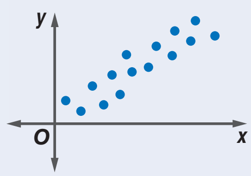
Measure your partner's height in inches. Then ask your partner to stand with his or her arms extended parallel to the floor. Measure the distance from the end of the longest finger on one hand to the longest finger on the other hand. Write these measures as the ordered pair (height, arm span) on the chalkboard.

1. Graph each of the ordered pairs listed on the chalkboard.
2. Examine the graph. Do you think there is a relationship between height and arm span? Explain.

The graph you made in the Mini Lab is called a scatter plot. A **scatter plot** is a graph that shows the relationship between two sets of data. In this type of graph, two sets of data are graphed as ordered pairs on a coordinate plane. Scatter plots often show a pattern, trend, or relationship between the variables.

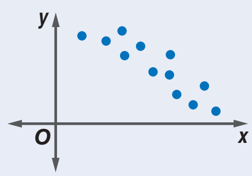
Concept Summary — **Types of Relationships**

Positive Relationship



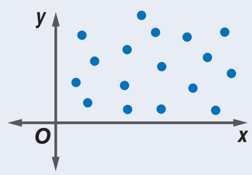
As x increases, y increases.

Negative Relationship



As x increases, y decreases.

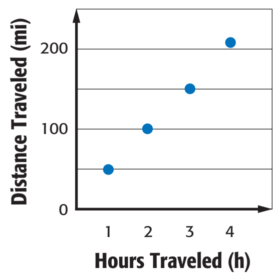
No Relationship



No obvious pattern

EXAMPLE Identify a Relationship

1 **Determine whether a scatter plot of the data for the hours traveled in a car and the distance traveled might show a *positive*, *negative*, or *no* relationship.**



As the number of hours you travel increases, the distance traveled increases. Therefore, the scatter plot shows a positive relationship.

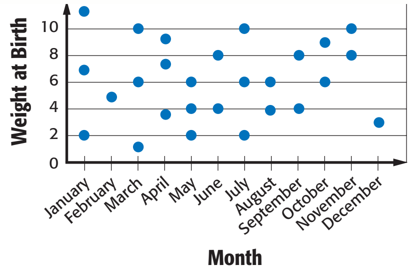
msmath3.net/extra_examples

Laura Sifferlin

EXAMPLE Identify a Relationship

2 **Determine whether a scatter plot of the data for the month of birth and birth weight show a *positive*, *negative*, or *no* relationship.**

Birth weight does not depend on the month of birth. Therefore, the scatter plot shows no relationship.



If a scatter plot shows a positive relationship or a negative relationship, a best-fit line can be drawn to represent the data. A **best-fit line** is a line that is very close to most of the data points.

REAL-LIFE MATH

LAKES The Great Lakes (Superior, Michigan, Huron, Erie, and Ontario) and their connecting waterways form the largest inland water transportation system in the world.

Source: *The World Book*

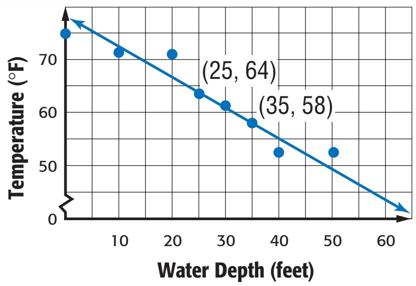
EXAMPLES Draw a Best-Fit Line

LAKES **The water temperatures at various depths in a lake are given.**

Water Depth (ft)	0	10	20	25	30	35	40	50
Temperature (°F)	74	72	71	64	61	58	53	53

3 **Make a scatter plot using the data. Then draw a line that seems to best represent the data.**

Graph each of the data points. Draw a line that best fits the data.



4 **Write an equation for this best-fit line.**

The line passes through points at (25, 64) and (35, 58). Use these points to find the slope of the line.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \dfrac{58 - 64}{35 - 25}$ $(x_1, y_1) = (25, 64)$, $(x_2, y_2) = (35, 58)$

$m = \dfrac{-6}{10}$ or $-\dfrac{3}{5}$ The slope is $-\dfrac{3}{5}$, and the y-intercept is 79.

Use the slope and y-intercept to write the equation.

$y = mx + b$ Slope-intercept form

$\downarrow \quad \downarrow$

$y = -\dfrac{3}{5}x + 79$ The equation for the best-fit line is $y = -\dfrac{3}{5}x + 79$.

5 **Use the equation to predict the temperature at a depth of 55 feet.**

$y = -\dfrac{3}{5}x + 79$ Equation for the best-fit line

$y = -\dfrac{3}{5}(55) + 79$ or 46 The temperature will be about 46°F.

STUDY TIP

Estimation Drawing a best-fit line using the method in this lesson is an estimation. Therefore, it is possible to draw different lines to approximate the same data.

Skill and Concept Check

1. **Writing Math** Describe how you can use a scatter plot to display two sets of related data.

2. **OPEN ENDED** Give an example of data that would show a negative relationship on a scatter plot.

3. **NUMBER SENSE** Suppose a scatter plot shows that as the values of x decrease, the values of y decrease. Does the scatter plot show a *positive*, *negative*, or *no* relationship?

GUIDED PRACTICE

Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship.

4. hours worked and earnings
5. miles per gallon and weight of car

EDUCATION **For Exercises 6–8, use the following table.**

Enrollment in U.S. Public and Private Schools (millions)

Year	Students	Year	Students	Year	Students
1900	15.5	1940	25.4	1980	41.7
1910	17.8	1950	25.1	1990	40.5
1920	21.6	1960	35.2	2000	46.9
1930	25.7	1970	45.6		

6. Draw a scatter plot of the data and draw a best-fit line.
7. Does the scatter plot show a *positive*, *negative*, or *no* relationship?
8. Use your graph to estimate the enrollment in public and private schools in 2010.

Data Update What is the current number of students in school? Visit msmath3.net/data_update to learn more.

Practice and Applications

Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship.

9. length of a side of a square and perimeter of the square
10. day of the week and amount of rain
11. grade in school and number of pets
12. length of time for a shower and amount of water used
13. outside temperature and amount of heating bill
14. age and expected number of years a person has yet to live
15. playing time and points scored in a basketball game
16. pages in a book and copies sold

HOMEWORK HELP

For Exercises	See Examples
9–16	1, 2
17–20	3–5

Extra Practice
See pages 644, 658.

FOOD **For Exercises 17–20, use the table.**

17. Draw a scatter plot of the data. Then draw a best-fit line.
18. Does the scatter plot show a *positive*, *negative*, or *no* relationship?
19. Write an equation for the best-fit line.
20. Use your equation to estimate the number of fat grams in a muffin with 350 Calories.

Nutritional Information of Commercial Muffins

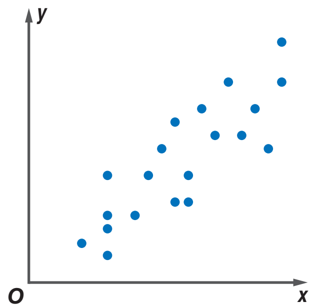
Muffin (brand)	Fat (grams)	Calories
A	2	250
B	3	300
C	4	260
D	9	220
E	14	410
F	15	390
G	10	300
H	18	430
I	23	480
J	20	490

21. **RESEARCH** Use the Internet or other resource to find the number of goals and assists for the players on one of the National Hockey teams for the past season. Make a scatter plot of the data.

22. **CRITICAL THINKING** A scatter plot of skateboard sales and swimsuit sales for each month of the year shows a positive relationship.
    a. Why might this be true?
    b. Does this mean that one factor caused the other? Explain.

Spiral Review with Standardized Test Practice

23. **MULTIPLE CHOICE** Find the situation that matches the scatter plot at the right.
    (A) adult height and year of birth
    (B) number of trees in an orchard and the number of apples produced
    (C) number of words written and length of pencil
    (D) length of campfire and amount of firewood remaining



24. **MULTIPLE CHOICE** What type of graph is most appropriate for displaying the change in house prices over several years?
    (F) scatter plot
    (G) circle graph
    (H) bar graph
    (I) line plot

State the slope and the y-intercept for the graph of each equation. (Lesson 11-5)

25. $y = \frac{4}{5}x + 7$
26. $y = -\frac{1}{6}x - 4$
27. $4x + y = 2$

28. **GEOMETRY** The vertices of a triangle are located at $(-2, 1)$, $(1, 7)$, and $(5, 1)$. Find the slope of each side of the triangle. (Lesson 11-4)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Graph each equation.** (Lessons 11-3 and 11-5)

29. $y = -x - 5$
30. $y = \frac{1}{2}x + 3$
31. $y = -\frac{1}{3}x + 1$
32. $y = 5x - 3$

11-6b Graphing Calculator Investigation

A Follow-Up of Lesson 11-6

What You'll LEARN

Use a graphing calculator to make a scatter plot.

Scatter Plots

You can use a TI-83/84 Plus graphing calculator to create scatter plots.

ACTIVITY

The following table gives the results of a survey listing the number of vehicles owned by a family and the average monthly gasoline cost in dollars. Make a scatter plot of the data.

Number of Vehicles	1	3	5	2	5	1	2
Monthly Gasoline Cost ($)	19	59	90	55	115	35	58

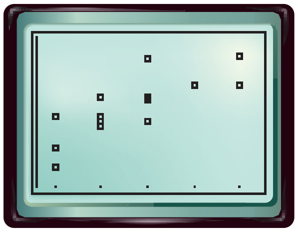
Number of Vehicles	2	1	3	4	3	3	2
Monthly Gasoline Cost ($)	80	62	77	90	80	112	63

STEP 1 **Clear the existing data.**
Keystrokes: STAT ENTER ▲ CLEAR ENTER

STEP 2 **Enter the data.**
Input each number of vehicles in L1 and press ENTER. Then enter the monthly gasoline cost in L2.

STEP 3 **Turn on the statistical plot.**
Select the scatter plot, L1 as the Xlist, and L2 as the Ylist.
Keystrokes: 2nd [STAT PLOT]
ENTER ENTER ▼ ENTER ▼
2nd [L1] ENTER 2nd [L2]
ENTER

STEP 4 **Graph the data.**
Keystrokes: ZOOM 9
Use the TRACE feature and the left and right arrow keys to move from one point to another.



EXERCISES

1. **Describe** the relationship of the data.
2. **RESEARCH** Find some data to use in a scatter plot. Enter the data in a graphing calculator. Determine whether the data has a *positive*, *negative*, or *no* relationship.

11-7 Graphing Systems of Equations

What You'll LEARN

Solve systems of linear equations by graphing.

NEW Vocabulary

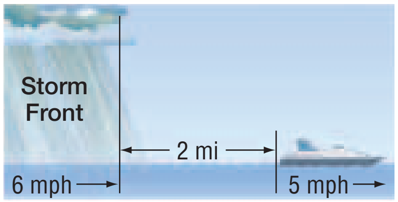
system of equations
substitution

REVIEW Vocabulary

solution: a value for the variable that makes an equation true (Lesson 1-8)

WHEN am I ever going to use this?

TRAVEL A storm is approaching a cruise ship. If x represents the number of hours, then $y = 6x$ represents the position of the storm, and $y = 5x + 2$ represents the position of the ship.



1. Graph both of the equations on a coordinate plane.
2. What are the coordinates of the point where the two lines intersect? What does this point represent?

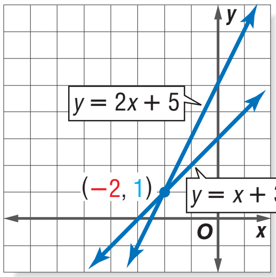
The equations $y = 6x$ and $y = 5x + 2$ form a system of equations. A set of two or more equations is called a **system of equations**.

When you find an ordered pair that is a solution of all of the equations in a system, you have solved the system. The ordered pair for the point where the graphs of the equations intersect is the solution.

EXAMPLE One Solution

1 Solve the system $y = x + 3$ and $y = 2x + 5$ by graphing.

The graphs of the equations appear to intersect at $(-2, 1)$. Check this estimate.



Check

$y = x + 3$	$y = 2x + 5$
$1 \stackrel{?}{=} -2 + 3$	$1 \stackrel{?}{=} 2(-2) + 5$
$1 = 1$ ✔	$1 = 1$ ✔

The solution of the system is $(-2, 1)$.

EXAMPLE Infinitely Many Solutions

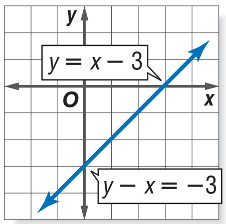
2 Solve the system $y = x - 3$ and $y - x = -3$ by graphing.

Write $y - x = -3$ in slope-intercept form.

$y - x = -3$ Write the equation.

$y - x + x = -3 + x$ Add x to each side.

$y = x - 3$ Both equations are the same.

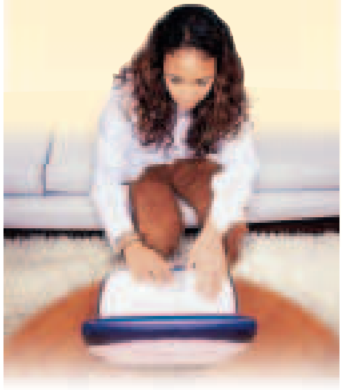


The solution of the system is all the coordinates of points on the graph of $y = x - 3$.

REAL-LIFE MATH

MONEY MATTERS The amount of online retail spending has been increasing in recent years. It more than doubled from 1999 with \$16.2 billion to 2000 with \$32.5 billion.

Source: BizRate.Com



EXAMPLE No Solution

3 **MONEY MATTERS The Buy Online Company charges \$1 per pound plus \$2 for shipping and handling. The Best Catalogue Company charges \$1 per pound plus \$3 for shipping and handling. For what weight will the shipping and handling for the two companies be the same?**

Let x equal the weight in pounds of the item or items ordered.

Let y equal the total cost of shipping and handling.

Write an equation to represent each company's charge for shipping and handling.

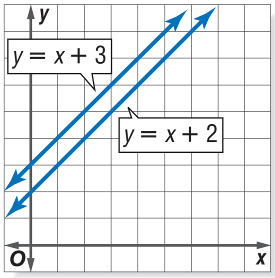
Buy Online Company: $y = 1x + 2$ or $y = x + 2$

Best Catalogue Company: $y = 1x + 3$ or $y = x + 3$

Graph the system of equations.

$y = x + 2$

$y = x + 3$



The graphs appear to be parallel lines. Since there is no coordinate pair that is a solution of both equations, there is no solution of this system of equations.

For any weight, the Buy Online Company will charge less than the Best Catalogue Company.

STUDY TIP

Slopes/Intercepts When the graphs of a system of equations have:
- different slopes, there is exactly one solution,
- the same slope and different y-intercepts, there is no solution,
- the same slope and the same y-intercept, there are many solutions.

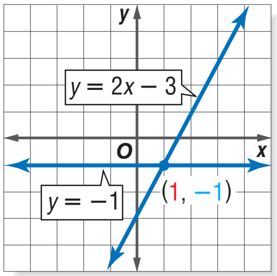
A more accurate way to solve a system of equations than by graphing is by using a method called **substitution**.

EXAMPLE Solve by Substitution

4 **Solve the system $y = 2x - 3$ and $y = -1$ by substitution.**

Since y must have the same value in both equations, you can replace y with -1 in the first equation.

$y = 2x - 3$	Write the first equation.
$-1 = 2x - 3$	Replace y with -1.
$-1 + 3 = 2x - 3 + 3$	Add 3 to each side.
$2 = 2x$	Simplify.
$\frac{2}{2} = \frac{2x}{2}$	Divide each side by 2.
$1 = x$	Simplify.



The solution of this system of equations is $(1, -1)$. You can check the solution by graphing. The graphs appear to intersect at $(1, -1)$, so the solution is correct.

Your Turn **Solve each system of equations by substitution.**

a. $y = x - 4$
$y = 7$

b. $y = 3x + 4$
$x = 2$

c. $y = -2x - 7$
$y = 1$

David Young-Wolff/PhotoEdit, Inc.

Skill and Concept Check

1. **Explain** what is meant by a system of equations and describe its solution.
2. **OPEN ENDED** Draw a graph of a system of equations that has $(-2, 3)$ as its solution.
3. **NUMBER SENSE** Describe the solution of the system $y = \frac{1}{2}x + 1$ and $y = \frac{1}{2}x - 3$ without graphing. Explain.

GUIDED PRACTICE

Solve each system of equations by graphing.

4. $y = 2x + 1$; $y = -x + 7$
5. $y = -2x + 4$; $y = 2x$
6. $y = 3x + 1$; $y = 3x - 1$

Solve each system of equations by substitution.

7. $y = 3x - 4$; $y = 8$
8. $y = -2x + 1$; $x = -3$
9. $y = 0.5x - 4$; $y = 1$

JOBS For Exercises 10–12, use the information at the right about the summer jobs of Neka and Savannah.

	Weekly Salary	Starting Bonus
Neka	\$300	\$200
Savannah	\$350	\$100

10. Write an equation for Neka's total income y after x weeks.
11. Write an equation for Savannah's total income y after x weeks.
12. When will Neka and Savannah have earned the same total amount? What will that amount be?

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
13–20	1–3
21–28	4

Extra Practice
See pages 644, 658.

Solve each system of equations by graphing.

13. $y = x - 4$; $y = -2x + 2$
14. $y = 2x - 3$; $y = -x - 6$
15. $y = 3x - 2$; $y = -\frac{1}{2}x + 5$
16. $y = \frac{1}{3}x + 1$; $y = -2x + 8$
17. $x + y = -3$; $x + y = 4$
18. $y - x = 0$; $2x + y = 3$
19. Graph the system $y = x + 8$ and $y = -2x - 1$. Find the solution.
20. Graph the system $y = -2x - 6$ and $y = -2x - 3$. Find the solution.

Solve each system of equations by substitution.

21. $y = 3x + 4$; $y = -5$
22. $y = -2x + 4$; $y = -6$
23. $y = -3x - 1$; $x = -4$
24. $y = 4x - 5$; $x = 2$
25. $y = -2x + 9$; $y = x$
26. $y = 5x - 8$; $y = x$
27. Solve the system $y = -3x + 5$ and $y = 2$ by substitution.
28. Solve the system $y = 2x - 1$ and $y = 5$ by substitution.

CLUBS For Exercises 29 and 30, use the following information.
The Science Club wants to order T-shirts for their members. The Shirt Shack will make the shirts for a \$30 set-up fee and then \$12 per shirt. T-World will make the same shirts for \$70 set-up fee and then \$8 per shirt.

29. For how many T-shirts will the cost be the same? What will be the cost?

30. If the club wants to order 30 T-shirts, which store should they choose?

HOT-AIR BALLOONS For Exercises 31–34, use the information at the right about two ascending hot-air balloons.

Balloon	Distance from Ground (meters)	Rate of Ascension (meters per minute)
A	60	15
B	40	20

31. Write an equation that describes the distance from the ground y of balloon A after x minutes.

32. Write an equation that describes the distance from the ground y of balloon B after x minutes.

33. When will the balloons be at the same distance from the ground?

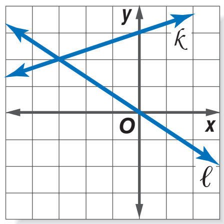
34. What is the distance of the balloons from the ground at that time?

35. CRITICAL THINKING One equation in a system of equations is $y = 2x + 1$.
- **a.** Write a second equation so that the system has (1, 3) as its only solution.
- **b.** Write an equation so that the system has no solutions.
- **c.** Write an equation so the system has many solutions.

Spiral Review with Standardized Test Practice

36. MULTIPLE CHOICE Which ordered pair represents the intersection of lines ℓ and k?

(A) $(-3, 2)$ (B) $(-2, 3)$ (C) $(3, -2)$ (D) $(2, -3)$



37. GRID IN The equation $c = 900 + 5t$ represents the cost c in cents that a long-distance telephone company charges for t minutes. Find the value of t if $c = 1{,}200$.

38. STATISTICS Determine whether a scatter plot of the speed of a car and the stopping distance would show a *positive*, *negative*, or *no* relationship. (Lesson 11-6)

Graph each equation using the slope and the y-intercept. (Lesson 11-5)

39. $y = -\frac{2}{3}x + 2$ **40.** $y = \frac{2}{5}x - 1$ **41.** $y = 4x - 3$

42. MULTI STEP Write an equation to represent *three times a number minus five is 16*. Then solve the equation. (Lesson 10-3)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Graph each inequality on a number line. (Lesson 10-5)

43. $x > -3$ **44.** $x \leq 5$ **45.** $x < 0$ **46.** $x \geq -1$

11-8 Graphing Linear Inequalities

What You'll LEARN

Graph linear inequalities.

NEW Vocabulary

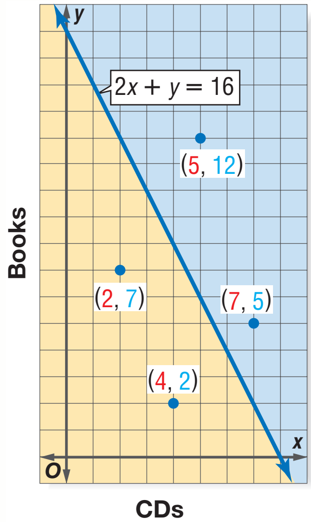
boundary
half plane

REVIEW Vocabulary

inequality: a mathematical sentence that contains $>$, $<$, $\neq$, $\leq$, or $\geq$ (Lesson 10-5)

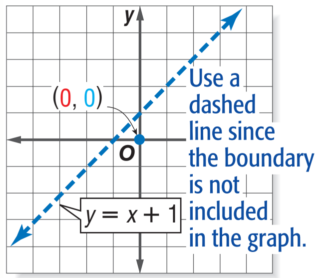
WHEN am I ever going to use this?

MONEY MATTERS At a sidewalk sale, one table has a variety of CDs for $2 each, and another table has a variety of books for $1 each. Sabrina wants to buy some CDs and books.



1. Use the graph at the right to list three different combinations of CDs and books that Sabrina can purchase for $16.
2. Suppose Sabrina wants to spend less than $16. Substitute (2, 7), (4, 2), (5, 12), and (7, 5) in $2x + y < 16$. Which values make the inequality true?
3. Which region do you think represents $2x + y < 16$?
4. Suppose Sabrina can spend more than $16. Substitute (2, 7), (4, 2), (5, 12), and (7, 5) in $2x + y > 16$. Which values make the inequality true?
5. Which region do you think represents $2x + y > 16$?

To graph an inequality such as $y < x + 1$, first graph the related equation $y = x + 1$. This is the **boundary**.



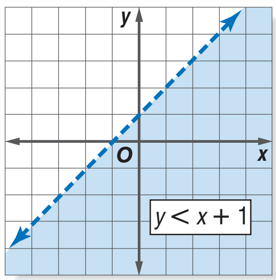
- If the inequality contains the symbol $\leq$ or $\geq$, a solid line is used to indicate that the boundary is included in the graph.
- If the inequality contains the symbol $<$ or $>$, a dashed line is used to indicate that the boundary is not included in the graph.

Next, test any point above or below the line to determine which region is the solution of $y < x + 1$. For example, it is easy to test (0, 0).

$y < x + 1$ Write the inequality.

$0 \stackrel{?}{<} 0 + 1$ Replace x with 0 and y with 0.

$0 < 1$ ✔ Simplify.

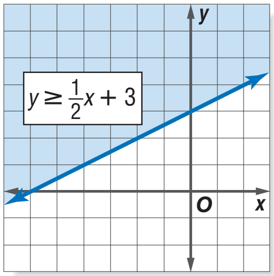


Since $0 < 1$ is true, (0, 0) is a solution of $y < x + 1$. Shade the region that contains this solution. This region is called a **half plane**. All points in this region are solutions of the inequality.

EXAMPLE Graph an Inequality

1 **Graph $y \geq \frac{1}{2}x + 3$.**

Step 1 Graph the boundary line $y = \frac{1}{2}x + 3$. Since $\geq$ is used in the inequality, make the boundary line a solid line.



Step 2 Test a point not on the boundary line, such as (0, 0).

$y \geq \frac{1}{2}x + 3$ Write the inequality.

$0 \overset{?}{\geq} \frac{1}{2}(0) + 3$ Replace *x* with 0 and *y* with 0.

$0 \ngeq 3$ Simplify.

Step 3 Since (0, 0) is not a solution of $y \geq \frac{1}{2}x + 3$, shade the region that does not contain (0, 0).

STUDY TIP

Check You may want to check the graph in Example 1 by choosing a point in the shaded region. Do the coordinates of that point make the inequality a true statement?

The solution of an inequality includes negative numbers as well as fractions. However, in real-life situations, sometimes negative numbers and fractions have no meaning.

EXAMPLE Graph an Inequality to Solve a Problem

2 **FAIRS** **At the local fair, rides cost \$3 and games cost \$1. Gloria has \$12 to spend on the rides and games. How can she spend her money?**

Let *x* represent the number of rides and *y* represent the number of games. Write an inequality.

Words	Cost of rides	plus	cost of games	is no more than	12.
Inequality	$3x$	$+$	$1y$	$\leq$	12

The related equation is $3x + y = 12$.

$3x + y = 12$ Write the equation.

$3x + y - 3x = 12 - 3x$ Subtract $3x$ from each side.

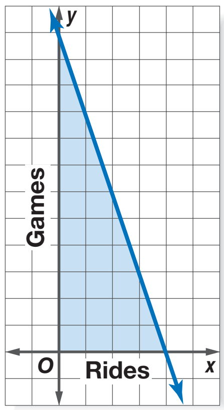
$y = -3x + 12$ Write in slope-intercept form.

Graph $y = -3x + 12$. Test (0, 0) in the original inequality.

$3x + 1y \leq 12$ Write the inequality.

$3(0) + 1(0) \overset{?}{\leq} 12$ Replace *x* with 0 and *y* with 0.

$0 \leq 12$ ✔ Simplify.



Since Gloria cannot ride or play a negative number of times or a fractional number of times, the answer is any pair of integers represented in the shaded region. For example, she could ride 3 rides and play 2 games.

REAL-LIFE MATH

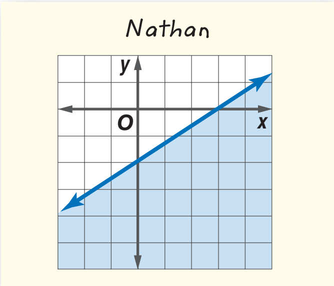
FAIRS Each year, there are more than 3,200 fairs held in the United States and Canada.

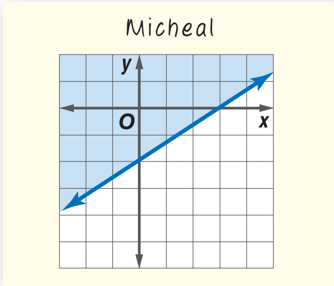
Source: *World Book*

Skill and Concept Check

1. **OPEN ENDED** Write an inequality that has a graph with a dashed line as its boundary. Graph the inequality.

2. **FIND THE ERROR** Nathan and Micheal are graphing $y \geq \frac{2}{3}x - 2$. Who is correct? Explain.





GUIDED PRACTICE

Graph each inequality.

3. $y > 2x - 1$
4. $y \leq \frac{4}{3}x - 2$
5. $y \geq \frac{1}{2}x$

GEOMETRY For Exercises 6 and 7, use the following information.
A formula for the perimeter of an isosceles triangle where x is the length of the legs and y is the length of the base is $P = 2x + y$.

6. Make a graph for all isosceles triangles that have a perimeter greater than 8 units.
7. Give the lengths of the legs and the base of three isosceles triangles with perimeters greater than 8 units.

Practice and Applications

Graph each inequality.

8. $y > x - 4$
9. $y \geq x + 5$
10. $y \leq -3x + 3$
11. $y < -x - 2$
12. $y > \frac{5}{2}x + 1$
13. $y < \frac{3}{4}x - 1$
14. $y \leq -\frac{3}{2}x + 2$
15. $y \geq -\frac{2}{5}x - 3$
16. $y < 5x + 3$
17. $y \leq 4x - 1$
18. $3x + y \geq 1$
19. $y - 2x > 6$
20. Graph the inequality *the sum of two numbers is less than 6.*
21. Graph the inequality *the sum of two numbers is greater than 4.*

SCHOOL For Exercises 22 and 23, use the following information.
Alberto must finish his math and social studies homework during the next 60 minutes.

22. Make a graph showing all the amounts of time Alberto can spend on each subject.
23. Give three possible ways Alberto can spend his time on math and social studies.

HOMEWORK HELP

For Exercises	See Examples
8–21	1
22–25	2

Extra Practice
See pages 645, 658.

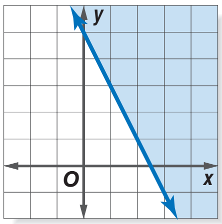
TRAVEL For Exercises 24 and 25, use the information below.
In the monetary system of the African country of Mauritania, five khoums equals one ouguiya. Heather is visiting Mauritania and wants to take at least an amount equal to 30 ouguiyas to the market. The inequality $\frac{1}{5}x + y \geq 30$, where x is the number of khoums and y is the number of ouguiyas, represents the situation.

24. Make a graph showing all the combinations of khoums and ouguiyas Heather can take to the market.
25. Give three possible ways Heather can take an appropriate amount to the market.
26. **CRITICAL THINKING** Graph the intersection of $y \leq -x - 3$ and $y > x + 2$.

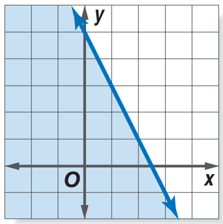
Spiral Review with Standardized Test Practice

27. **MULTIPLE CHOICE** Which is the graph of $2x + y > 5$?

(A)



(B)



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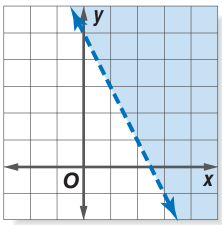

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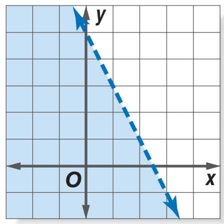

28. **MULTIPLE CHOICE** Which ordered pair is *not* a solution of $y + 3 \leq 2x$?

(F) (1, 2) (G) (2, 1) (H) (3, −2) (I) (2, −3)

Solve each system of equations by graphing. (Lesson 11-7)

29. $y = 2x - 5$
 $y = -x + 1$
30. $y = x + 2$
 $y = -x$
31. $y = -2x + 4$
 $y = -2x - 2$
32. $y = -x + 4$
 $y = 2x - 2$

33. **STATISTICS** Determine whether a scatter plot of the amount of studying and test scores would show a *positive*, *negative*, or *no* relationship. (Lesson 11-6)

Find the area of each figure. (Lesson 7-1)

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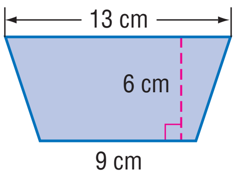

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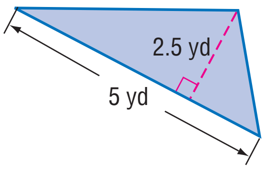

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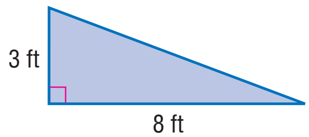


37. **MULTI STEP** The original price of a jacket is \$58. Find the price of the jacket if it is marked down 25%. (Lesson 5-7)

CHAPTER 11 Study Guide and Review

Vocabulary and Concept Check

- arithmetic sequence (p. 512)
- best-fit line (p. 540)
- boundary (p. 548)
- common difference (p. 512)
- common ratio (p. 513)
- dependent variable (p. 518)
- domain (p. 518)
- function (p. 517)
- function table (p. 518)
- geometric sequence (p. 513)
- half plane (p. 548)
- independent variable (p. 518)
- linear function (p. 523)
- range (p. 518)
- scatter plot (p. 539)
- sequence (p. 512)
- slope formula (p. 526)
- slope-intercept form (p. 533)
- substitution (p. 545)
- system of equations (p. 544)
- term (p. 512)
- *x*-intercept (p. 523)
- *y*-intercept (p. 523)

Choose the correct term or number to complete each sentence.

1. The (domain, range) is the set of input values of a function.
2. The range is the set of (input, output) values of a function.
3. A (sequence, term) is an ordered list of numbers.
4. A geometric sequence has a (common difference, common ratio).
5. A(n) (arithmetic sequence, geometric sequence) has a common difference.
6. The (x-intercept, y-intercept) has the coordinates $(0, b)$.
7. The (half-plane, boundary) is the graph of the equation related to an inequality.
8. The slope formula is $m = \left(\frac{y_2 - y_1}{x_2 - x_1}, \frac{x_2 - x_1}{y_2 - y_1}\right)$.

Lesson-by-Lesson Exercises and Examples

11-1 Sequences (pp. 512–515)

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio. Write the next three terms of the sequence.

9. 64, 32, 16, 8, 4, …
10. −7, −4, −1, 2, 5, …
11. 1, 2, 6, 24, 120, …
12. 1, −1, 1, −1, 1, …
13. **SAVINGS** Loretta has \$5 in her piggy bank. Each week, she adds \$1.50. If she does not take any money out of the piggy bank, how much will she have after 6 weeks?

Example 1 **State whether the sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio. Write the next three terms of the sequence.**

1, −2, 4, −8, 16, …

$$1 \xrightarrow{\times(-2)} -2 \xrightarrow{\times(-2)} 4 \xrightarrow{\times(-2)} -8 \xrightarrow{\times(-2)} 16, \ldots$$

The terms have a common ratio of −2, so the sequence is geometric. The next three terms are 16(−2) or −32, −32(−2) or 64, and 64(−2) or −128.

11-2 Functions (pp. 517–520)

Find each function value.

14. $f(3)$ if $f(x) = 3x + 1$

15. $f(9)$ if $f(x) = 1 - 3x$

16. $f(0)$ if $f(x) = 2x + 6$

17. $f(-11)$ if $f(x) = -2x$

18. $f(-2)$ if $f(x) = x - 1$

19. $f(2)$ if $f(x) = \frac{1}{2}x - 4$

Example 2 Complete the function table for $f(x) = 2x - 1$.

x	$2x - 1$	$f(x)$
-2	$2(-2) - 1$	-5
0	$2(0) - 1$	-1
1	$2(1) - 1$	1
5	$2(5) - 1$	9

11-3 Graphing Linear Functions (pp. 522–525)

Graph each function.

20. $y = -2x + 1$

21. $y = x - 4$

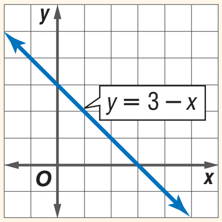
22. $y = -3x$

23. $y = \frac{1}{2}x - 2$

24. GEOMETRY The function $y = 4x$ represents the perimeter y of a square with side x units long. Graph $y = 4x$.

Example 3 Graph $y = 3 - x$.

x	$3 - x$	y	(x, y)
-1	$3 - (-1)$	4	$(-1, 4)$
0	$3 - 0$	3	$(0, 3)$
2	$3 - 2$	1	$(2, 1)$
3	$3 - 3$	0	$(3, 0)$



11-4 The Slope Formula (pp. 526–529)

Find the slope of each line that passes through each pair of points.

25. $A(-2, 3)$, $B(-1, 5)$

26. $E(-3, 2)$, $F(-3, 5)$

27. $G(6, 2)$, $H(1, 5)$

28. $K(2, 1)$, $L(-3, 1)$

Example 4 Find the slope of the line that passes through $A(-3, 2)$ and $B(5, -1)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Definition of slope

$m = \frac{-1 - 2}{5 - (-3)}$ or $-\frac{3}{8}$ $(x_1, y_1) = (-3, 2)$, $(x_2, y_2) = (5, -1)$

11-5 Slope-Intercept Form (pp. 533–536)

State the slope and y-intercept for the graph of each equation.

29. $y = 2x + 5$

30. $y = \frac{1}{2}x - 7$

31. $y = \frac{1}{5}x + 6$

32. $y = -3x - 2$

33. $y = -\frac{3}{4}x + 7$

34. $y = 3x + 7$

Example 5 State the slope and y-intercept of the graph of $y = -\frac{1}{2}x + 3$.

$y = -\frac{1}{2}x + 3$ Write the equation.

$y = mx + b$

The slope of the graph is $-\frac{1}{2}$, and the y-intercept is 3.

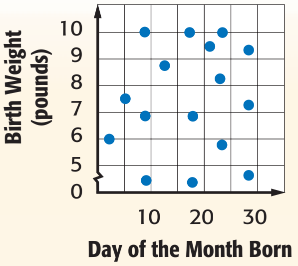
Mixed Problem Solving
For mixed problem-solving practice, see page 658.

11-6 Statistics: Scatter Plots (pp. 539–542)

Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship.

35. number of people in the household and the cost of groceries
36. day of the week and temperature
37. child's age and grade level in school
38. temperature outside and amount of clothing

Example 6 **Determine whether the graph at the right shows a *positive*, *negative*, or *no* relationship.**



Since there is no obvious pattern, there is no relationship.

11-7 Graphing Systems of Equations (pp. 544–547)

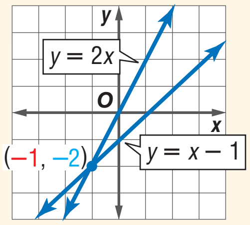
Solve each system of equations by graphing.

39. $y = 2x$, $y = x + 1$
40. $y = 3x - 1$, $y = x - 3$
41. $y = 2x - 2$, $y = x + 2$
42. $y = x - 4$, $y = -x + 2$

Solve each system of equations by substitution.

43. $y = -2x + 7$, $x = 3$
44. $y = 3x - 5$, $x = 4$

Example 7 **Solve the system of equations $y = 2x$ and $y = x - 1$ by graphing.**



The graphs of the equation appear to intersect at $(-1, -2)$.
Check this estimate.

11-8 Graphing Linear Inequalities (pp. 548–551)

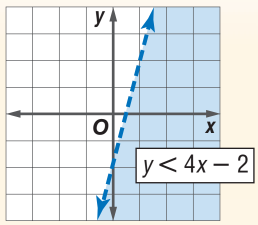
Graph each inequality.

45. $y > x$
46. $y < -2x$
47. $y \geq 2x + 3$
48. $y \leq -x + 5$
49. $y > 3x + 5$
50. $y \leq 2x + 1$

51. **FESTIVALS** At the Spring Festival, games cost \$2 and rides cost \$3. Nate wants to spend no more than \$20 at the festival. Give three possible ways Nate can spend his money.

Example 8 **Graph $y < 4x - 2$.**

Graph the boundary line $y = 4x - 2$. Since the $<$ symbol is used in the inequality, make the boundary line a dashed line.



Test a point not on the boundary line such as (0, 0). Since (0, 0) is not a solution of $y < 4x - 2$, shade the region that does not contain (0, 0).

CHAPTER 11 Practice Test

Vocabulary and Concepts

1. **Describe** how you can tell that there is no solution when you graph a system of equations.

2. **Describe** two different ways to graph $y = 2x + 5$.

Skills and Applications

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is *arithmetic* or *geometric*, state the common difference or common ratio. Then write the next three terms of the sequence.

3. $-10, -6, -2, 2, 6, \ldots$

4. $\frac{1}{5}, 1, 5, 25, 125, \ldots$

5. $61, 50, 40, 31, 23, \ldots$

Find each function value.

6. $f(-2)$ if $f(x) = \frac{x}{2} + 5$

7. $f(3)$ if $f(x) = -2x + 6$

Graph each function or inequality.

8. $y = \frac{1}{3}x - 1$

9. $y = -4x + 1$

10. $y < -2x + 3$

11. $y \geq x - 5$

Find the slope of the line that passes through each pair of points.

12. $A(-2, 5), B(-2, 1)$

13. $C(0, 3), D(-5, 2)$

14. $E(2, -1), F(5, -3)$

CHILD CARE For Exercises 15–17, use the following information.
The cost for a child to attend a certain day care center is \$35 a day plus a registration fee of \$50. The cost y for x days of child care is $y = 35x + 50$.

15. Graph the equation.

16. What does the y-intercept represent?

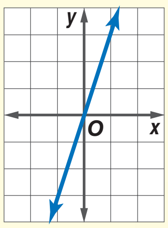
17. What does the slope of the graph represent?

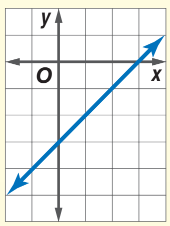
18. Solve the system $y = x + 1$ and $y = 2x - 2$ by graphing.

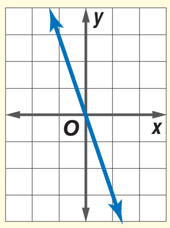
19. **TRAVEL** Would a scatter plot of data describing the gallons of gas used and the miles driven show a *positive*, *negative*, or *no* relationship?

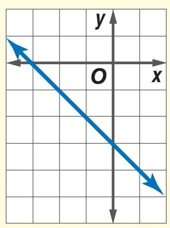
Standardized Test Practice

20. **MULTIPLE CHOICE** Which is the graph of $y = -3x$?

A


B


C


D


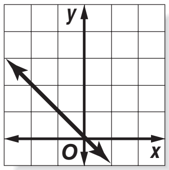
Chapter 11 Standardized Test Practice

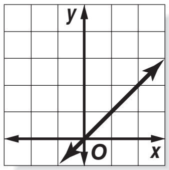
PART 1 Multiple Choice

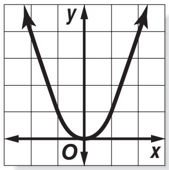
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

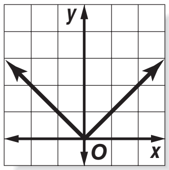
1. If the following ordered pairs are plotted on a graph, which graph passes through all five points? (Prerequisite Skill, p. 614)

   $(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)$

   A 

   B 

   C 

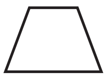
   D 

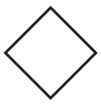
2. Which could be the value of x if $0.6 < x < 68\%$? (Lesson 5-2)

   F $\frac{3}{5}$ G $\frac{2}{3}$

   H $\frac{5}{7}$ I $\frac{6}{8}$

3. Which of the following is *not* a quadrilateral? (Lesson 6-4)

   A  B 

   C  D 

TEST-TAKING TIP

Question 3 Read each question carefully so that you do not miss key words such as *not* or *except*. Then read every answer choice carefully. If allowed to write in the test booklet, cross off each answer choice that you know is not the answer, so you will not consider it again.

4. What is the value of $f(x)$ when $x = 5$? (Lesson 11-1)

x	$f(x)$
0	3
1	4
2	5
5	?

   F 7 G 8

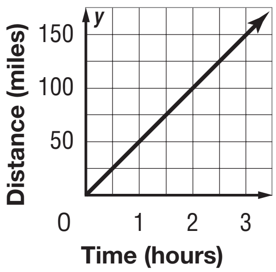
   H 9 I 10

5. Which of the following ordered pairs is a solution of $y = \frac{1}{2}x - 4$? (Lesson 11-2)

   A $(-2, -3)$ B $(4, 2)$

   C $(6, 1)$ D $(8, 0)$

6. The graph shows the distance Kimberly has traveled. What does the slope of the graph represent? (Lesson 11-5)



   F the distance Kimberly traveled

   G how long Kimberly has traveled

   H Kimberly's average speed

   I the time Kimberly will arrive

7. Which display would be most appropriate for the data in a table that shows the relationship between height and weight of 20 students? (Lesson 11-6)

   A scatter plot B line plot

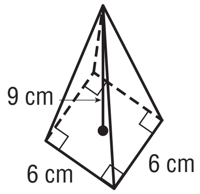
   C bar graph D circle graph

PART 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. On a number line, how many units apart are -6 and 7? (Lesson 1-3)

9. Find the volume of the pyramid. (Lesson 7-6)



10. Mr. Thomas is drawing names out of a hat in order to create class debating teams. There are 13 girls and 12 boys in the class, and the first name picked is a girl's name. What is the probability that a boy's name will be drawn next? (Lesson 8-5)

11. Molly received grades of 79, 92, 68, 90, 72, and 92 on her history tests. What measure of central tendency would give her the highest grade for the term? (Lesson 9-4)

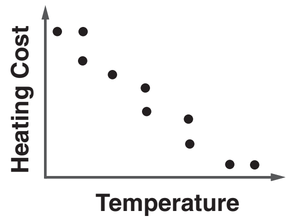
12. A pair of designer jeans costs \$98, which is \$35 more than 3 times the cost of a discount store brand. The equation to find the cost of the discount store brand d is $3d + 35 = 98$. What is the price of the discount store brand of jeans? (Lesson 10-3)

13. Copy and complete the function table for $f(x) = 0.4x + 2$. (Lesson 11-2)

x	y
-5	
0	
5	
10	

14. Graph $y = \frac{1}{2}x - 2$. (Lesson 11-3)

15. Describe the relationship shown in the graph. (Lesson 11-6)



16. What ordered pair is a solution of both $y = 3x - 5$ and $y = x - 7$? (Lesson 11-7)

PART 3 Extended Response

Record your answers on a sheet of paper. Show your work.

17. Study the data below. (Lesson 11-6)

Date	Number of Customers	Ice Cream Scoops Sold
June 1	75	100
June 2	125	230
June 3	350	460
June 4	275	370
June 5	175	300
June 6	225	345
June 7	210	325

a. What type of display would be most appropriate for this data?

b. Graph the data.

c. Describe the relationship of the data.

18. In Major League Soccer, a team gets 3 points for a win and 1 point for a tie. (Lesson 11-8)

a. Write an inequality for the number of ways a team can earn more than 10 points.

b. Graph the inequality.

c. Compare the values graphed and those that actually satisfy the situation.