CHAPTER 12

Algebra: Nonlinear Functions and Polynomials

558–559 Duomo/CORBIS

What does racing have to do with math?

As a race car increases its speed, or accelerates, the distance it travels each second also increases. The relationship between these two quantities, however, is not linear. The distance d that a race car travels in time t, given the rate of acceleration a, can be described by the quadratic function $d = \frac{1}{2}at^2$.

You will solve a problem about racing in Lesson 12-2.

GETTING STARTED

Diagnose Readiness

Take this quiz to see if you are ready to begin Chapter 12. Refer to the lesson number in parentheses for review.

Vocabulary Review

Choose the correct term to complete each sentence.

1. A function in which the graph of the solutions forms a line is called a (straight, linear) function. (Lesson 11-3)
2. The (base, exponent) is the number in a power that is multiplied. (Lesson 2-8)

Prerequisite Skills

Identify the like terms in each expression. (Lesson 10-1)

3. $3x + 5 - x$
4. $2 - 4n + 1 + 6n$

Rewrite each expression using parentheses so that the like terms are grouped together. (Lessons 1-2 and 10-1)

5. $(a + 2b) + (2a - 5b)$
6. $(8w + 7x) + (3w + 9x)$

Rewrite each expression as an addition expression by using the additive inverse. (Lessons 1-4 and 1-5)

7. $3 - 5y$
8. $2m - 7n$

Write each expression using exponents. (Lesson 2-8)

9. $6 \cdot 6 \cdot 6 \cdot 6$
10. $3 \cdot 7 \cdot 7 \cdot 3 \cdot 7$

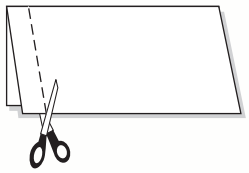
Use the Distributive Property to rewrite each expression. (Lesson 10-1)

11. $9(d + 2)$
12. $-8(f - 3)$

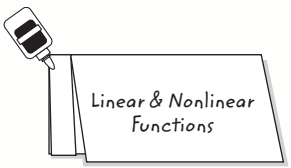
FOLDABLES Study Organizer

Nonlinear Functions Make this Foldable to organize your notes. Begin with 7 sheets of $8\frac{1}{2}$" × 11" paper.

STEP 1 **Fold and Cut** Fold a sheet of paper in half lengthwise. Cut a 1" tab along the left edge through one thickness.



STEP 2 **Glue and Label** Glue the 1" tab down. Write the title of the lesson on the front tab.



STEP 3 **Repeat and Staple** Repeat Steps 1–2 for the remaining sheets of paper. Staple together to form a booklet.

Noteables™

Chapter Notes Each time you find this logo throughout the chapter, use your *Noteables™: Interactive Study Notebook with Foldables™* or your own notebook to take notes. Begin your chapter notes with this Foldable activity.

Math nline

Readiness To prepare yourself for this chapter with another quiz, visit **msmath3.net/chapter_readiness**

12-1 Linear and Nonlinear Functions

What You'll LEARN

Determine whether a function is linear or nonlinear.

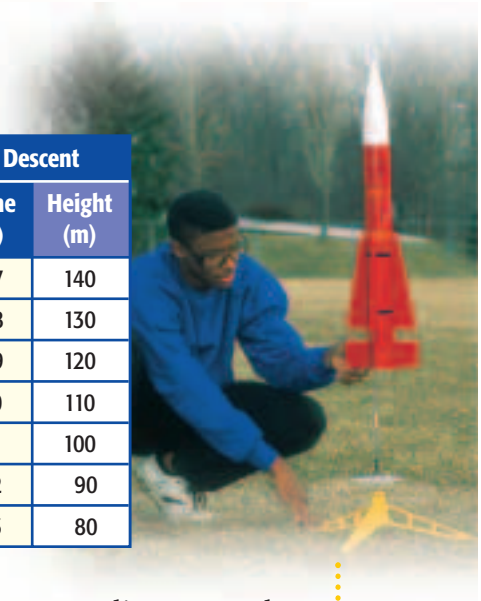
NEW Vocabulary

nonlinear function

REVIEW Vocabulary

function: a relationship where one quantity depends upon another (Lesson 11-2)

WHEN am I ever going to use this?



ROCKETRY The tables show the flight data for a model rocket launch. The first table gives the rocket's height at each second of its ascent, or upward flight. The second table gives its height as it descends back to Earth using a parachute.

Ascent	
Time (s)	Height (m)
0	0
1	38
2	74
3	106
4	128
5	138
6	142

Descent	
Time (s)	Height (m)
7	140
8	130
9	120
10	110
11	100
12	90
13	80

1. During its ascent, did the rocket travel the same distance each second? Explain.
2. During its descent, did the rocket travel the same distance each second? Explain.
3. Graph the data whose ordered pairs are (time, height) for the rocket's ascent and descent on separate axes. Connect the points with a straight line or smooth curve. Then compare the two graphs.

In Lesson 11-3, you learned that linear functions have graphs that are straight lines. These graphs represent constant rates of change. **Nonlinear functions** do not have constant rates of change. Therefore, their graphs are not straight lines.

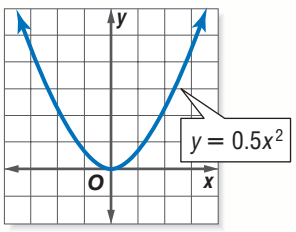
STUDY TIP

Nonlinear Functions
The function in Example 1 is a *quadratic function*. The function in Example 2 is an *exponential function*.

EXAMPLES Identify Functions Using Graphs

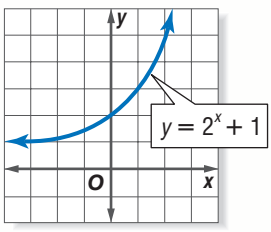
Determine whether each graph represents a *linear* or *nonlinear* function. Explain.

1



The graph is a curve, not a straight line. So it represents a nonlinear function.

2



This graph is also a curve. So it represents a nonlinear function.

Doug Martin

CONTENTS

STUDY TIP

Identifying Linear Equations Always examine an equation after it has been solved for y to see that the power of x is 1 or 0. Then check to see that x does not appear in the denominator.

Since the equation for a linear function can be written in the form $y = mx + b$, where m represents the constant rate of change, you can determine whether a function is linear by examining its equation.

EXAMPLES Identify Functions Using Equations

Determine whether each equation represents a *linear* or *nonlinear* function. Explain.

3 $y = x + 4$

Since the equation can be written as $y = 1x + 4$, this function is linear.

4 $y = \frac{6}{x}$

Since x is in the denominator, the equation cannot be written in the form $y = mx + b$. So this function is nonlinear.

A nonlinear function does not increase or decrease at the same rate. You can use a table to determine if the rate of change is constant.

EXAMPLES Identify Functions Using Tables

Determine whether each table represents a *linear* or *nonlinear* function. Explain.

5

	x	y	
+2	2	50	−15
+2	4	35	−15
+2	6	20	−15
	8	5	

As x increases by 2, y decreases by 15 each time. The rate of change is constant, so this function is linear.

6

	x	y	
+3	1	1	+15
+3	4	16	+33
+3	7	49	+51
	10	100	

As x increases by 3, y increases by a greater amount each time. The rate of change is not constant, so this function is nonlinear.

REAL-LIFE MATH

BASKETBALL The NCAA women's basketball tournament begins with 64 teams and consists of 6 rounds of play.

7 **BASKETBALL** **Use the table to determine whether the number of teams is a linear function of the number of rounds of play.**

Round(s) of play	Teams
1	2
2	4
3	8
4	16
5	32

Examine the differences between the number of teams for each round.

$4 - 2 = 2 \quad 8 - 4 = 4 \quad 16 - 8 = 8 \quad 32 - 16 = 16$

While there is a pattern in the differences, they are not the same. Therefore, this function is nonlinear.

Your Turn **Determine whether each equation or table represents a *linear* or *nonlinear* function. Explain.**

a. $y = 2x^3 + 1$

b. $y = 3x$

c.

x	0	5	10	15
y	20	16	12	8

Elise Amendola/AP/Wide World Photos

Skill and Concept Check

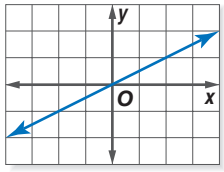
1. **OPEN ENDED** Give an example of a nonlinear function using a table of values.

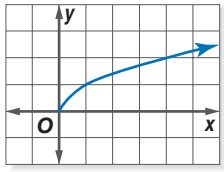
2. **Which One Doesn't Belong?** Identify the function that is not linear. Explain your reasoning.

$y = 2x$	$y = x^2$	$y - 2 = x$	$x - y = 2$

GUIDED PRACTICE

Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

3. 

4. 

5. $y = \frac{x}{3}$

6. $y - x = 1$

7.

x	3	6	9	12
y	12	10	8	6

8.

x	1	2	3	4
y	1	4	9	16

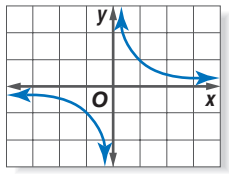
Practice and Applications

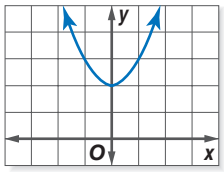
HOMEWORK HELP

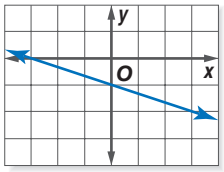
For Exercises	See Examples
9–14	1, 2
15–22	3, 4
23–28	5, 6
29–31	7

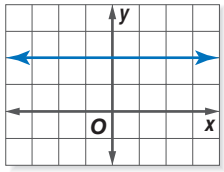
Extra Practice
See pages 645, 659.

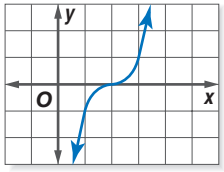
Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

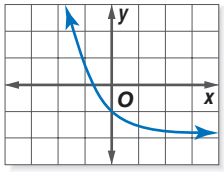
9. 

10. 

11. 

12. 

13. 

14. 

15. $xy = -9$

16. $y = 0.6x$

17. $y = x^3 - 1$

18. $y = 4x^2 + 9$

19. $y = 2^x$

20. $y = \frac{4}{x}$

21. $y = 7$

22. $y = \frac{3x}{2}$

23.

x	1	2	3	4
y	0	2	6	12

24.

x	−1	0	1	2
y	−4	1	6	11

25.

x	2	5	8	11
y	21	19	17	15

26.

x	−4	0	4	8
y	2	1	−1	−4

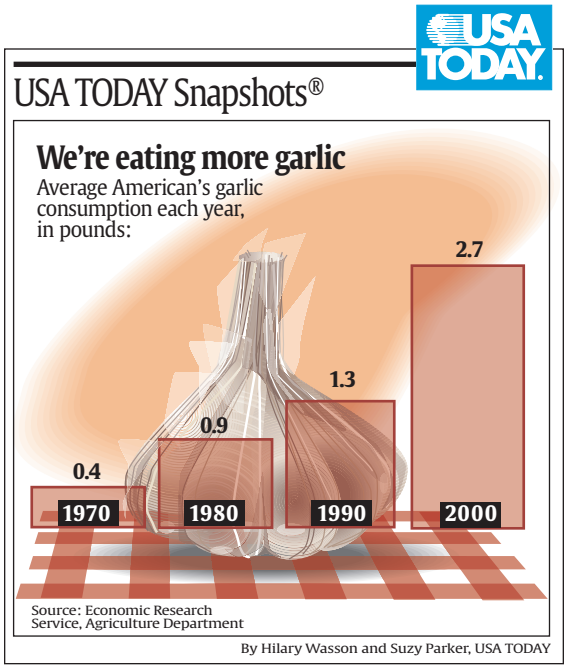
27.

x	4	6	8	10
y	4	13.5	32	62.5

28.

x	0.5	1	1.5	2
y	15	8	1	−6

29. **FOOD** The graphic shows the increase in garlic consumption from 1970 to 2000. Would you describe the growth as linear or nonlinear? Explain.



Data Update Is the growth in the consumption of your favorite food linear or nonlinear? Visit msmath3.net/data_update to learn more.

GEOMETRY For Exercises 30 and 31, use the following information.
Recall that the circumference of a circle is equal to pi times its diameter and that the area of a circle is equal to pi times the square of its radius.

30. Is the circumference of a circle a linear or nonlinear function of its diameter? Explain.

31. Is the area of a circle a linear or nonlinear function of its radius? Explain.

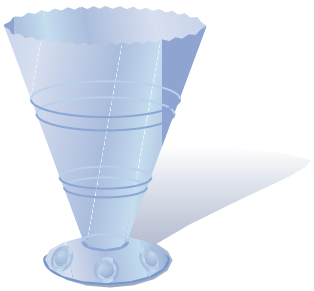
32. **CRITICAL THINKING** *True* or *False*? All graphs of straight lines are linear functions. Explain your reasoning or provide a counterexample.

Spiral Review with Standardized Test Practice

33. **MULTIPLE CHOICE** Which equation represents a nonlinear function?

(A) $y = 3x + 1$ (B) $y = \frac{x}{3}$ (C) $2xy = 10$ (D) $y = 3(x - 5)$

34. **SHORT RESPONSE** Water is poured at a constant rate into the vase at the right. Draw a graph of the water level as a function of time. Is the water level a linear or nonlinear function of time? Explain your reasoning.



COPYING For Exercises 35 and 36, use the following information.
Black-and-white copies at Copy Express cost \$0.12 each, and color copies cost \$1.00 each. Suppose you want to spend no more than \$10 on copies of your club's flyers. (Lesson 11-8)

35. Write an inequality to represent this situation.

36. Graph the inequality and use the graph to determine three possible combinations of copies you could make.

Solve each system of equations by substitution. (Lesson 11-7)

37. $y = 2x + 1$, $y = 3$

38. $y = -4x - 3$, $y = 1$

39. $y = -5x + 8$, $y = -2$

40. $y = 0.5x - 6$, $y = -4$

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Graph each function. (Lesson 11-3)

41. $y = 2x$

42. $y = x + 3$

43. $y = 3x - 2$

44. $y = \frac{1}{3}x + 1$

12-2a Graphing Calculator Investigation

A Preview of Lesson 12-2

What You'll LEARN

Use a graphing calculator to graph families of quadratic functions.

Families of Quadratic Functions

In Lesson 11-5a, you discovered that families of linear functions share the same slope or y-intercept. Families of nonlinear functions also share a common characteristic. You can use a TI-83/84 Plus graphing calculator to investigate families of quadratic functions.

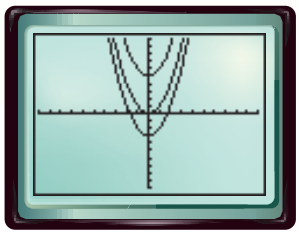
ACTIVITY

Graph $y = x^2$, $y = x^2 + 5$, and $y = x^2 - 3$ on the same screen.

STEP 1 Clear any existing equations from the Y= list.
Keystrokes: [Y=] [CLEAR]

STEP 2 Enter each equation.
Keystrokes: [X,T,θ,n] [x^2] [ENTER]
[X,T,θ,n] [x^2] [+] 5 [ENTER]
[X,T,θ,n] [x^2] [−] 3 [ENTER]

STEP 3 Graph the equations in the standard viewing window.
Keystrokes: [ZOOM] 6



EXERCISES

1. **Compare and contrast** the three equations you graphed.
2. **Describe** how the graphs of the three equations are related.
3. **MAKE A CONJECTURE** How does changing the value of c in the equation $y = x^2 + c$ affect the graph?
4. **Use a graphing calculator** to graph $y = 0.5x^2$, $y = x^2$, and $y = 2x^2$.
5. **Compare and contrast** the three equations you graphed in Exercise 4.
6. **Describe** how the graphs of the three equations are related.
7. **MAKE A CONJECTURE** How does changing the value of a in the equation $y = ax^2$ affect the graph?
8. **Write** a family of three quadratic functions. Describe the common characteristic of their graphs.

12-2 Graphing Quadratic Functions

What You'll LEARN

Graph quadratic functions.

NEW Vocabulary

quadratic function

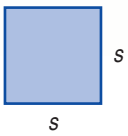
HANDS-ON Mini Lab

Materials

- graph paper

Work with a partner.

You know that the area A of a square is equal to the length of a side s squared, $A = s^2$. What happens to the area of a square as its side length is increased?



s	s^2	(s, A)
0	0	(0, 0)
1	1	(1, 1)
2		
3		
4		
5		
6		

STEP 1 Copy and complete the table.

STEP 2 Graph the ordered pairs from the table. Connect them with a smooth curve.

1. Is the relationship between the side length and the area of a square linear or nonlinear? Explain.
2. Describe the shape of the graph.

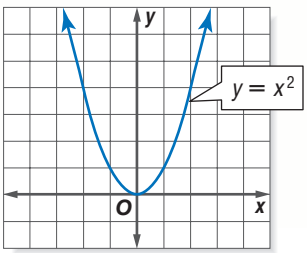
A **quadratic function** is a function in which the greatest power of the variable is 2.

EXAMPLES Graph Quadratic Functions: $y = ax^2$

1 Graph $y = x^2$.

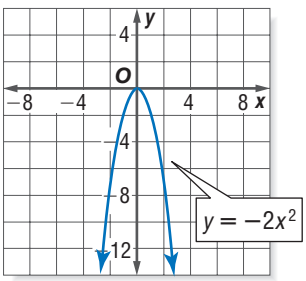
To graph a quadratic function, make a table of values, plot the ordered pairs, and connect the points with a smooth curve.

x	x^2	y	(x, y)
-2	$(-2)^2 = 4$	4	$(-2, 4)$
-1	$(-1)^2 = 1$	1	$(-1, 1)$
0	$(0)^2 = 0$	0	(0, 0)
1	$(1)^2 = 1$	1	(1, 1)
2	$(2)^2 = 4$	4	(2, 4)



2 Graph $y = -2x^2$.

x	$-2x^2$	y	(x, y)
-2	$-2(-2)^2 = -8$	-8	$(-2, -8)$
-1	$-2(-1)^2 = -2$	-2	$(-1, -2)$
0	$-2(0)^2 = 0$	0	(0, 0)
1	$-2(1)^2 = -2$	-2	$(1, -2)$
2	$-2(2)^2 = -8$	-8	$(2, -8)$



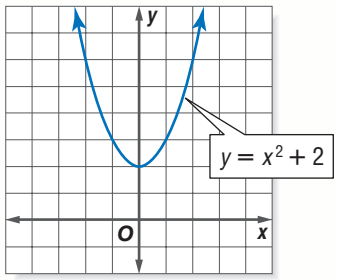
STUDY TIP

Quadratic Functions
The graph of a quadratic function is called a *parabola*.

EXAMPLES Graph Quadratic Functions: $y = ax^2 + c$

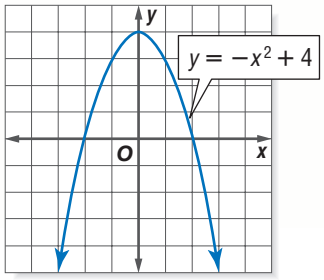
3 Graph $y = x^2 + 2$.

x	$x^2 + 2$	y	(x, y)
-2	$(-2)^2 + 2 = 6$	6	$(-2, 6)$
-1	$(-1)^2 + 2 = 3$	3	$(-1, 3)$
0	$(0)^2 + 2 = 2$	2	$(0, 2)$
1	$(1)^2 + 2 = 3$	3	$(1, 3)$
2	$(2)^2 + 2 = 6$	6	$(2, 6)$



4 Graph $y = -x^2 + 4$.

x	$-x^2 + 4$	y	(x, y)
-2	$-(-2)^2 + 4 = 0$	0	$(-2, 0)$
-1	$-(-1)^2 + 4 = 3$	3	$(-1, 3)$
0	$-(0)^2 + 4 = 4$	4	$(0, 4)$
1	$-(1)^2 + 4 = 3$	3	$(1, 3)$
2	$-(2)^2 + 4 = 0$	0	$(2, 0)$



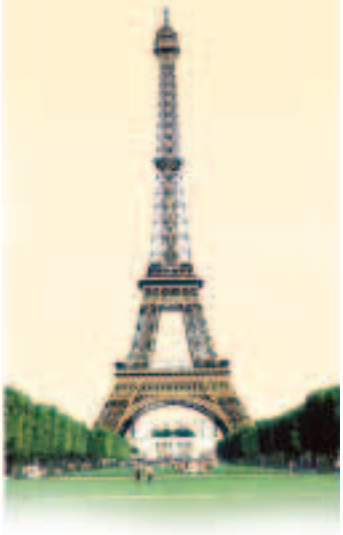
Your Turn **Graph each function.**

a. $y = x^2 + 1$ b. $y = -2x^2 - 1$ c. $y = -x^2$

REAL-LIFE MATH

MONUMENTS The Eiffel Tower in Paris, France, opened in 1889 as part of the World Exposition. It is about 986 feet tall.

Source: www.structurae.de



Lance Nelson/CORBIS

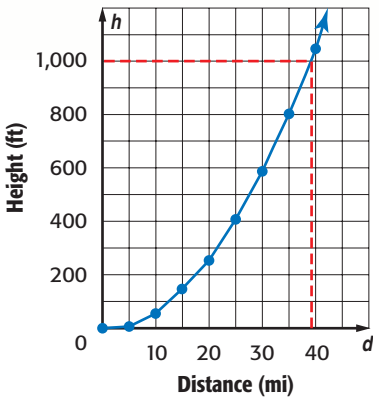
Many real-life situations can be described using quadratic functions.

EXAMPLE Graph a Function to Solve a Problem

5 **MONUMENTS** **The function $h = 0.66d^2$ represents the distance d in miles you can see from a height of h feet. Graph this function. Then use your graph and the information at the left to estimate how far you could see from the top of the Eiffel Tower.**

The equation $h = 0.66d^2$ is quadratic, since the variable d has an exponent of 2. Distance cannot be negative, so use only positive values of h.

d	$h = 0.66d^2$	(d, h)
0	$0.66(0)^2 = 0$	$(0, 0)$
5	$0.66(5)^2 = 16.5$	$(5, 16.5)$
10	$0.66(10)^2 = 66$	$(10, 66)$
15	$0.66(15)^2 = 148.5$	$(15, 148.5)$
20	$0.66(20)^2 = 264$	$(20, 264)$
25	$0.66(25)^2 = 412.5$	$(25, 412.5)$
30	$0.66(30)^2 = 594$	$(30, 594)$
35	$0.66(35)^2 = 808.5$	$(35, 808.5)$
40	$0.66(40)^2 = 1{,}056$	$(40, 1{,}056)$



At a height of 986 feet, you could see approximately 39 miles.

CONTENTS

Skill and Concept Check

1. Writing Math Explain how to determine whether a function is quadratic.

2. **OPEN ENDED** Write a quadratic function of the form $y = ax^2 + c$ and explain how to graph it.

3. **Which One Doesn't Belong?** Identify the function whose graph does not have the same characteristic as the other three. Explain your reasoning.

$y = 2x^2 + 1$	$y = -5x^2$	$y = 7x - 3$	$y = 4x^2 - 2$

GUIDED PRACTICE

Graph each function.

4. $y = 3x^2$
5. $y = -5x^2$
6. $y = 0.5x^2$
7. $y = x^2 - 2$
8. $y = -x^2 + 1$
9. $y = -2x^2 + 2$

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
10–13	1, 2
14–23	3, 4
24–29	5

Extra Practice
See pages 645, 659.

Graph each function.

10. $y = 4x^2$
11. $y = -3x^2$
12. $y = -1.5x^2$
13. $y = 3.5x^2$
14. $y = x^2 + 6$
15. $y = x^2 - 4$
16. $y = 2x^2 - 1$
17. $y = 2x^2 + 3$
18. $y = -x^2 + 2$
19. $y = -x^2 - 5$
20. $y = -4x^2 - 1$
21. $y = -3x^2 + 2$

22. Graph the function $y = 0.5x^2 + 1$.
23. Graph the function $y = \frac{1}{3}x^2 - 2$.

RACING **For Exercises 24–26, use the following information.**

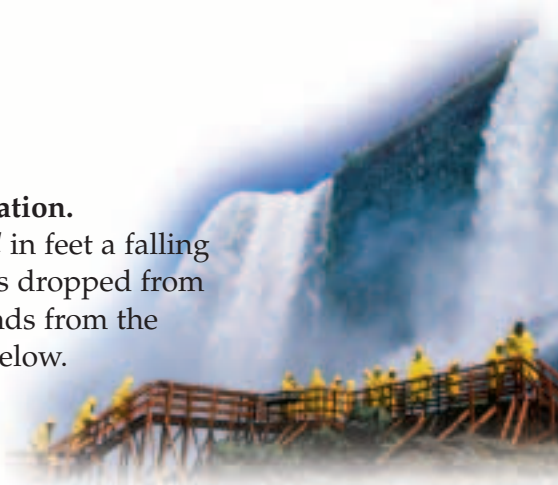
The function $d = \frac{1}{2}at^2$ represents the distance d that a race car will travel over an amount of time t given the rate of acceleration a. Suppose a car is accelerating at a rate of 5 feet per second every second.

24. Graph $d = \frac{1}{2}(5t^2)$.
25. Find the distance traveled after 10 seconds.
26. About how long would it take the car to travel 125 feet?

WATERFALL **For Exercises 27–29, use the following information.**

The quadratic equation $d = -16t^2 + h$ models the distance d in feet a falling object is from the ground or other surface t seconds after it is dropped from a beginning height of h feet. Suppose a drop of water descends from the 182-foot tall American Falls in New York, toward the river below.

27. Graph $d = -16t^2 + 182$.
28. How high is a drop of water after 2 seconds?
29. After about how many seconds will the drop of water reach the river below?



GEOMETRY **For Exercises 30 and 31, write a function for each of the following. Then graph the function in the first quadrant.**

30. the volume V of a cube as a function of the edge length a

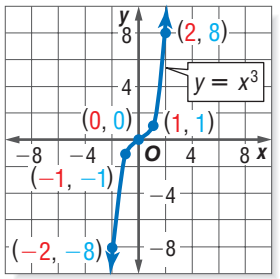
31. the volume V of a rectangular prism as a function of a fixed height of 5 and a square base of varying length s

CRITICAL THINKING The graphs of quadratic functions may have exactly one highest point, called a *maximum*, or exactly one lowest point, called a *minimum*.

Graph each quadratic equation. Determine whether each graph has a maximum or a minimum. If so, give the coordinates of each point.

32. $y = 2x^2 + 1$ 33. $y = -x^2 + 5$ 34. $y = x^2 - 3$

EXTENDING THE LESSON Another type of nonlinear function is graphed at the right. A *cubic function*, such as $y = x^3$, is a function in which the greatest power is 3.



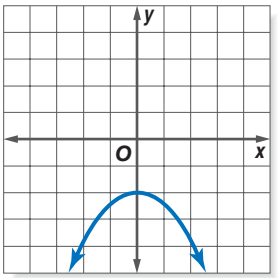
Graph each function. (*Hint*: You may need to let x represent decimal values.)

35. $y = 2x^3$ 36. $y = x^3 + 1$ 37. $y = 2x^3 + 2$

38. Graph the equations $y = x^2$ and $y = x^3$ on the same coordinate plane. Describe their similarities and differences.

Spiral Review with Standardized Test Practice

39. **MULTIPLE CHOICE** Which equation represents the graph at the right?



(A) $y = 2x^2 - 2$ (B) $y = -0.5x^2 - 2$

(C) $y = -x^2 + 2$ (D) $y = x^2 - 2$

40. **MULTIPLE CHOICE** Which equation represents a quadratic function?

(F) $y = 2x$ (G) $y = \frac{2}{x}$

(H) $y = x + 2$ (I) $y = -x^2 + 8$

Determine whether each equation represents a *linear* or *nonlinear* function. (Lesson 12-1)

41. $y = x - 5$ 42. $y = 3x^3 + 2$ 43. $x + y = -6$ 44. $y = -2x^2$

Graph each inequality. (Lesson 11-8)

45. $y < 2x$ 46. $y \geq x + 1$ 47. $y > -x - 3$ 48. $y \leq -3x + 4$

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Identify the like terms in each expression.** (Lesson 10-1)

49. $4a + 1 - 2a$ 50. $2x + 3x + 5 - 1$ 51. $-1 - 2d + 3 + d$ 52. $x + 2 - 7x + 8$

12-3a HANDS-ON LAB

A Preview of Lesson 12-3

Modeling Expressions with Algebra Tiles

What You'll LEARN

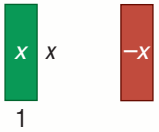
Model expressions using algebra tiles.

Materials

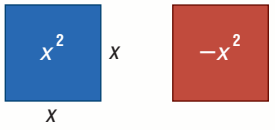
- algebra tiles

In a set of algebra tiles, the integer 1 is represented by a tile that is 1 unit by 1 unit. Notice that the area of this tile is 1 square unit. The opposite of 1, -1, is represented by a red tile with the same shape and size.

The variable x is represented by a tile that is 1 unit by x units. Notice that the area of this tile is x square units. The opposite of x, $-x$, is represented by a red tile with the same shape and size.



Similarly, the expression x^2 is represented by a tile that is x units by x units. A red tile with the same shape and size is used to represent $-x^2$.



You can use these tiles to model expressions like $2x^2 + 5x - 6$.

ACTIVITY

Use algebra tiles to model $2x^2 + 5x - 6$.

$2x^2 + 5x - 6$

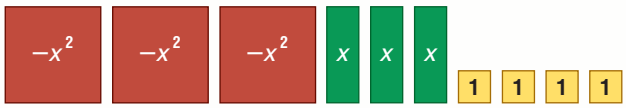
Your Turn **Use algebra tiles to model each expression.**

a. $4x^2$ b. $-3x^2$ c. $3x^2 - 4x$ d. $-x^2 + 2x$

e. $x^2 - x + 1$ f. $-2x^2 + x - 5$ g. $2x^2 - 3x + 2$ h. $-4x^2 + 3x + 8$

Writing Math

1. **Name** the expression modeled below.



2. **MAKE A CONJECTURE** What might a model of the expression x^3 look like?

12-3 Simplifying Polynomials

What You'll LEARN

Simplify polynomials.

NEW Vocabulary

monomial
polynomial

REVIEW Vocabulary

like terms: terms that contain the same variable (Lesson 10-1)
simplest form: an algebraic expression that has no like terms or parentheses (Lesson 10-1)

READING in the Content Area

For strategies in reading this lesson, visit **msmath3.net/reading**.

WHEN am I ever going to use this?

MONEY Suppose you need money to buy a drink and a snack. The table shows the number and type of coins you find in your backpack and in your pocket.

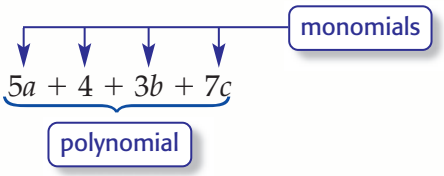
Coin Type	Number in Backpack	Number in Pocket
Quarter	3	0
Dime	5	2
Nickel	2	3
Penny	4	0

1. Let q, d, n, and p represent the value of a quarter, a dime, a nickel, and a penny, respectively. Write an expression for the total amount of money in your backpack.
2. Write an expression for the total amount of money in your pocket.
3. Write an expression for the total amount of money in all.

In Lesson 10-1, you learned that like terms, such as $-5d$ and $2d$, can be combined using the Distributive Property.

$$-5d + 2d = (-5 + 2)d \quad \text{Distributive Property}$$
$$= -3d \quad \text{Simplify.}$$

The terms of an expression are also called monomials. A **monomial** is a number, a variable, or a product of numbers and/or variables. An algebraic expression that is the sum or difference of one or more monomials is called a **polynomial**.



You have already learned how to simplify polynomials like $3x + 4 + 2x - 8$ by combining like terms. You can use the same process to simplify polynomials containing more than one variable.

EXAMPLE 1 Simplify a Polynomial

Simplify $-5d + 2n + 4d - 3n$.

The like terms in this expression are $-5d$ and $4d$, and $2n$ and $-3n$.

$-5d + 2n + 4d - 3n$ Write the polynomial.
$= -5d + 2n + 4d + (-3n)$ Definition of subtraction
$= (-5d + 4d) + [2n + (-3n)]$ Group like terms.
$= -1d + (-1n)$ or $-d - n$ Simplify by combining like terms.

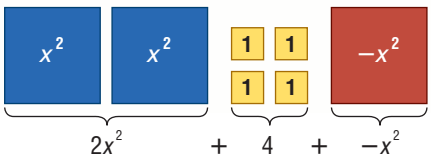
The expression $2x^2$ is another example of a monomial, since it is the product of 2, x, and x. You can simplify expressions like $2x^2 + 4 - x^2$ using algebra tiles.

EXAMPLE 2 Simplify Polynomials

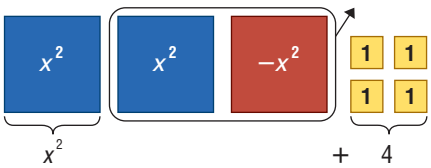
Simplify $2x^2 + 4 - x^2$.

Use the definition of subtraction to write this polynomial as $2x^2 + 4 + (-x^2)$.

Method 1 Use models.



Group tiles with the same shape and remove zero pairs.



Thus, $2x^2 + 4 - x^2 = x^2 + 4$.

Method 2 Use symbols.

Write the polynomial. Then group and add like terms.

$$\begin{aligned} &2x^2 + 4 + (-x^2) \\ &= [2x^2 + (-x^2)] + 4 \\ &= [2x^2 + (-1x^2)] + 4 \\ &= 1x^2 + 4 \\ &= x^2 + 4 \end{aligned}$$

Look Back To review **zero pairs**, see Lesson 1-4.

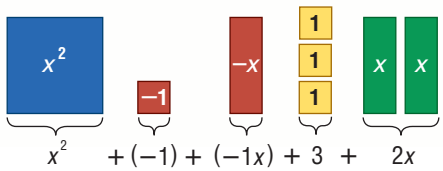
From these examples, you can see that like terms must have the same variable and the same power. Thus, $2x^2$ and $3x^2$ are like terms, while $4x^2$ and $5x$ are not.

EXAMPLE 3 Simplify Polynomials

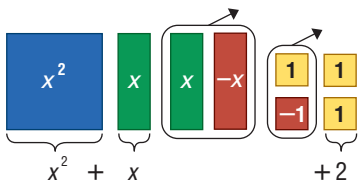
Simplify $x^2 - 1 - x + 3 + 2x$.

$x^2 - 1 - x + 3 + 2x$ is equal to $x^2 + (-1) + (-1x) + 3 + 2x$.

Method 1 Use models.



Group tiles with the same shape and remove zero pairs.



Thus, $x^2 - 1 - x + 3 + 2x = x^2 + x + 2$.

Method 2 Use symbols.

Write the polynomial. Then group and add like terms.

$$\begin{aligned} &x^2 + (-1) + (-1x) + 3 + 2x \\ &= x^2 + (-1x + 2x) + (-1 + 3) \\ &= x^2 + 1x + 2 \\ &= x^2 + x + 2 \end{aligned}$$

Standard Form When simplifying polynomials, it is customary to write the result in *standard form*; that is, with the powers of the variable decreasing from left to right.

$-5x^2 + 3x + 2$, *not* $3x - 5x^2 + 2$

Skill and Concept Check

1. **OPEN ENDED** Write a polynomial with four terms that simplifies to $5a - 9b$.
2. **Writing Math** Explain why $6x$ and $3x^2$ are not like terms.
3. **Which One Doesn't Belong?** Identify the expression that is not a like term. Explain your reasoning.

$2y^2$	$-x^2$	$5x^2$	$-4x^2$

GUIDED PRACTICE

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*.

4. $4c + 5d + 6d + c$
5. $-7x - 8 - 2y$
6. $9g - 9h + 3g + 1$
7. $4x + x^2 + 2x$
8. $-x^2 + 5 + 3x - 1 + x^2$

9. $-5w^2 + 3w^2 - 8w$
10. $-9m^2 + 4m - m + 2$
11. $6g^2 + 5 - 7g + 3 - 8g^2$

Practice and Applications

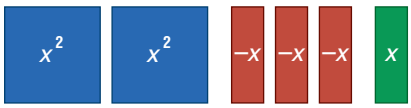
Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*.

HOMEWORK HELP

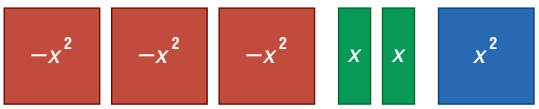
For Exercises	See Examples
12–17, 36	1
18–35, 37	2, 3

Extra Practice See pages 646, 659.

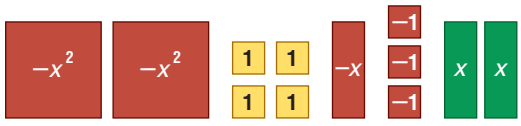
12. $6a + 8b - 7a + b$
13. $5x + 7y + 8 - z$
14. $-n + 4p + 5 - 6n$
15. $3f - 2g - 9g + 5f$
16. $-8c - d + 4c + 2$
17. $2j + 7 + k - 9$
18. $2x^2 - 3x + x$



19. $-3x^2 + 2x + x^2$



20. $-2x^2 + 4 - x - 3 + 2x$



21. $x^2 + x + 2 - x^2 - 3x - 5$

22. $m^2 + m - 3$
23. $a + 5a^2 - 7a$
24. $4 - 3x^2 + 6x + x^2$
25. $2w^2 - 6w - w + 1$
26. $3k^2 + 4 - 8k + k - 2$
27. $y^2 + 8y + 1 + 7y^2 - 4$
28. $a^2 + 3a^2 - 4a + a - 7 - 1$
29. $-z^2 + z^2 - 5z + 9z - 2 + 13$
30. $b^2 + 6b - 9 + b^2 - b + 3$
31. $r^2 - 3r + 8 + 2r^2 - 4r + 4$
32. $11 - 4n^3 - 8 + n^3 - 4n^3$
33. $-5t^3 - 8t^2 + 4t - 6 + 7t^3 + 3t$
34. $1.4x^2 - 3.8x + 1.2x^2 + 4.5x$
35. $\frac{3}{4}y^2 - 5y - \frac{1}{4}y + 5y$

36. **COOKIES** The table shows the number of boxes of each type of cookie Orlando and Emma bought from Science Club members. If m represents the cost of mint cookies, p the cost of peanut butter cookies, and c the cost of chocolate chip cookies per box, write an expression in simplest form for the total amount spent by Orlando and Emma on cookies.

Name	Mint	Peanut Butter	Chocolate Chip
Orlando	2 boxes	1 box	0 boxes
Emma	0 boxes	2 boxes	3 boxes

37. **SAVINGS** Shanté receives \$50 each birthday from her aunt. Her parents put this money in a savings account with an interest rate of r. The table gives the account balance after each birthday. Write the balance of Shanté's account after her third birthday in simplest form.

Birthday	Balance (\$)
1	50
2	$(50r + 50) + 50$
3	$(50r^2 + 100r + 50) + (50r + 50) + 50$

38. **CRITICAL THINKING** Determine whether $2x^2 + 3x = 5x^2$ is *sometimes*, *always*, or *never* true for all x. Explain your reasoning.

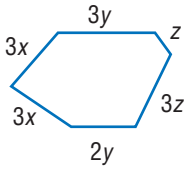
Spiral Review with Standardized Test Practice

39. **MULTIPLE CHOICE** Simplify $x^2 - 4x - 5x + 3 - 2x^2 + 9$.

(A) $3x^2 - 9x + 12$ (B) $2x^2$

(C) $-2x^2 - 9x + 12$ (D) $-x^2 - 9x + 12$

40. **MULTIPLE CHOICE** Write the perimeter of the figure in simplest form.



(F) $14xyz$ (G) $15xyz$

(H) $6x + 5y + 3z$ (I) $6x + 5y + 4z$

Graph each function. (Lesson 12-2)

41. $y = 5x^2$ 42. $y = x^2 + 5$ 43. $y = x^2 - 4$ 44. $y = -x^2 - 3$

45. **BIOLOGY** The table shows how long it took for the first 400 bacteria cells to grow in a petri dish. Is the growth of the bacteria a linear function of time? Explain. (Lesson 12-1)

Time (min)	46	53	57	60
Number of Cells	100	200	300	400

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Rewrite each expression using parentheses so that the like terms are grouped together.** (Lessons 1-2 and 10-1)

46. $(a + 2) + (3a + 4)$ 47. $(2n + 5) + (5n + 1)$

48. $(c + d) + (7c - 2d)$ 49. $(x^2 + 4x) + (6x^2 - 8x)$

12-4 Adding Polynomials

What You'll LEARN

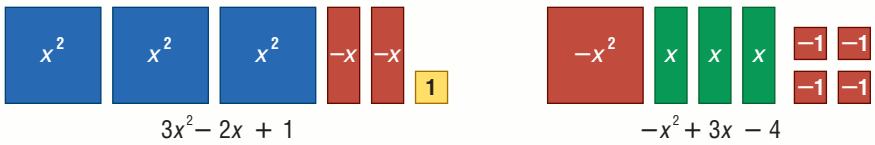
Add polynomials.

HANDS-ON Mini Lab

Materials
- algebra tiles

Work with a partner.

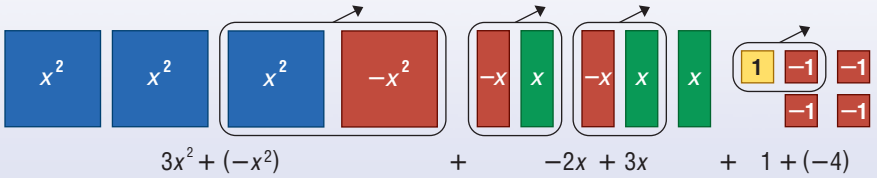
Consider the polynomials $3x^2 - 2x + 1$ and $-x^2 + 3x - 4$ modeled below.



Follow these steps to add the polynomials.

STEP 1 Combine the tiles that have the same shape.

STEP 2 Remove any zero pairs.



1. Write the polynomial for the tiles that remain.
2. Use algebra tiles to find $(x^2 + x - 2) + (6x^2 - 5x - 1)$.

You can add polynomials horizontally or vertically by combining like terms.

EXAMPLES Add Polynomials

1 **Find $(4x + 1) + (2x + 3)$.**

Method 1 Add vertically.

$$\begin{array}{r} 4x + 1 \\ (+)\ 2x + 3 \\ \hline 6x + 4 \end{array}$$

Align like terms.

Add.

Method 2 Add horizontally.

$$\begin{aligned} &(4x + 1) + (2x + 3) \\ &= (4x + 2x) + (1 + 3) \\ &= 6x + 4 \end{aligned}$$

Associative and Commutative Properties

The sum is $6x + 4$.

2 **Find $(3x^2 + 5x - 9) + (x^2 + x + 6)$.**

Method 1 Add vertically.

$$\begin{array}{r} 3x^2 + 5x - 9 \\ (+)\ x^2 + x + 6 \\ \hline 4x^2 + 6x - 3 \end{array}$$

Method 2 Add horizontally.

$$\begin{aligned} &(3x^2 + 5x - 9) + (x^2 + x + 6) \\ &= (3x^2 + x^2) + (5x + x) + (-9 + 6) \\ &= 4x^2 + 6x - 3 \end{aligned}$$

The sum is $4x^2 + 6x - 3$.

STUDY TIP

Adding Vertically
When adding vertically, be sure to correctly identify the terms of each polynomial. For example, the last term of the polynomial $2x^2 - 3$ is -3, not 3.

EXAMPLES Add Polynomials

3 **Find $(7y^2 + 2y) + (-5y + 8)$.**

$(7y^2 + 2y) + (-5y + 8) = 7y^2 + (2y - 5y) + 8$ Group like terms.

$= 7y^2 - 3y + 8$ Simplify.

The sum is $7y^2 - 3y + 8$.

4 **Find $(6x^2 - x + 5) + (2x^2 - 3)$.**

$$\begin{array}{r} 6x^2 - x + 5 \\ (+)\ 2x^2 \quad\ - 3 \\ \hline 8x^2 - x + 2 \end{array}$$

Leave a space because there is no other term like $-x$.

The sum is $8x^2 - x + 2$.

Your Turn **Add.**

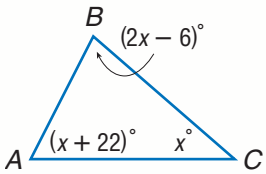
a. $(4x + 3) + (x - 1)$ **b.** $(10a^2 + 5a + 7) + (a^2 - 3)$

Polynomials are often used to represent measures of geometric figures.

Standardized Test Practice

EXAMPLE Use Polynomials to Solve a Problem

5 **MULTIPLE-CHOICE TEST ITEM** Find the measure of $\angle B$ in the figure at the right.



Ⓐ 41° Ⓑ 63° Ⓒ 76° Ⓓ 166°

Read the Test Item The figure is a triangle. The sum of the measures of the angles of a triangle equals 180°. The measure of each angle is determined by the value of x.

Solve the Test Item

Write an equation to find the value of x.

The sum of the measures of the angles	equals	180	
$(2x - 6) + (x + 22) + x$	$=$	180	Write the equation.
$(2x + x + x) + (-6 + 22) =$		180	Group like terms.
$4x + 16 =$		180	Simplify.
$-16 =$		-16	Subtract 16 from each side.
$4x =$		164	Simplify.
$\frac{4x}{4} =$		$\frac{164}{4}$	Divide each side by 4.
$x =$		41	Simplify.

Find the measure of angle B.

$m\angle B = 2x - 6$ Write the expression for the measure of angle B.

$= 2(41) - 6$ Replace x with 41.

$= 82 - 6$ or 76 Simplify.

The measure of $\angle B$ is 76°. The answer is C.

Test-Taking Tip

Many standardized tests provide a list of common geometry facts and formulas. Be sure to find this list before the test begins so you can refer to it easily.

Skill and Concept Check

1. **OPEN ENDED** Write two polynomials whose sum is $4x - 5y$.

2. **FIND THE ERROR** Benito and Cleavon are adding $5a^2 - 7a$ and $3a^2 + 2$. Who is correct? Explain.

Benito

$$\begin{array}{r} 5a^2 - 7a \\ (+)\ 3a^2 + 2 \\ \hline 8a^2 - 5a \end{array}$$

Cleavon

$$\begin{array}{r} 5a^2 - 7a \\ (+)\ 3a^2 \quad\quad + 2 \\ \hline 8a^2 - 7a + 2 \end{array}$$

GUIDED PRACTICE

Add.

3. $\begin{array}{r} h + 3 \\ (+)\ 2h + 1 \\ \hline \end{array}$

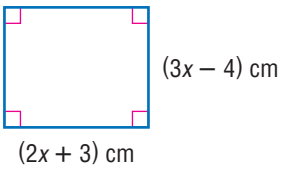
4. $\begin{array}{r} 2b^2 + 6b + 9 \\ (+)\ b^2 + 2b - 7 \\ \hline \end{array}$

5. $\begin{array}{r} 4t^2 + t + 1 \\ (+)\ 3t^2 \quad\quad - 5 \\ \hline \end{array}$

6. $(6g^2 - g + 3) + (-2g^2 - 3g + 1)$

7. $(8f^2 - 3f) + (5f - 9)$

GEOMETRY For Exercises 8–10, use the rectangle at the right.



8. Write an expression in simplest form for the perimeter of the rectangle.
9. Find the value of x if the perimeter of the rectangle is 48.
10. Find the measure of the length and the width of the rectangle.

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
11–30	1–4
39–40	5

Extra Practice
See pages 646, 659.

Add.

11. $\begin{array}{r} 5y + 6 \\ (+)\ 2y + 4 \\ \hline \end{array}$

12. $\begin{array}{r} 5p^2 + 3 \\ (+)\ 8p^2 + 1 \\ \hline \end{array}$

13. $\begin{array}{r} s^2 - s - 4 \\ (+)\ 4s^2 + 2s - 5 \\ \hline \end{array}$

14. $\begin{array}{r} k^2 + 6k - 2 \\ (+)\ 7k^2 - 3k - 1 \\ \hline \end{array}$

15. $\begin{array}{r} 4m^2 + m - 5 \\ (+)\ 3m^2 \quad\quad + 9 \\ \hline \end{array}$

16. $\begin{array}{r} 8x^2 - 6x - 7 \\ (+)\ -4x^2 - 6x \\ \hline \end{array}$

17. $(2c - 4) + (3c + 3)$

18. $(9z + 6) + (-5z - 6)$

19. $(7j^2 + j + 1) + (j^2 - 5j - 2)$

20. $(4q^2 - 2q - 1) + (q^2 + 5q + 1)$

21. $(5d^2 - 6) + (3d^2 + 5)$

22. $(-9w^2 + 4) + (4w^2 - 9)$

23. $(4n^2 + 8) + (2n^2 - 5n + 1)$

24. $(-6r - 2) + (r^2 + 9r + 4)$

25. $(5v^2 - v + 1) + (v^2 + v + 1)$

26. $(6x^2 - 5x - 4) + (-x^2 - 8x - 9)$

27. $(5m^2 - 2) + (4m + 6)$

28. $(-3g - 10) + (6g^2 + 7g)$

29. $(-2b^2 - 3b - 7) + (5b + 2)$

30. $(-3a^2 - 2a - 9) + (-3a^2 - 5a - 3)$

Add. Then evaluate each sum if $x = 6$, $y = 3$, and $z = -5$.

31. $(6x + 2y) + (-4x - y)$

32. $(-3y + 5z) + (10y - 2z)$

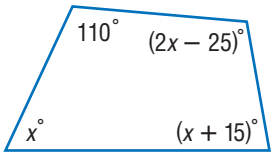
33. $(-3x + 4z) + (5y - 2z)$

34. $(4x - 6y - 13z) + (-3x - 4y + 11z)$

WORK **For Exercises 35–38, use the following information.**
Wei-Ling works at a grocery store a few hours after school on weekdays and baby-sits on weekends. She makes the same hourly wage for both jobs. During one week, Wei-Ling worked 18 hours at the grocery store, and $9 was deducted for taxes. She worked 7 hours baby-sitting, and no taxes were deducted. Let x represent her hourly pay.

35. Write a polynomial expression to represent Wei-Ling's grocery store pay.
36. Write a polynomial expression to represent Wei-Ling's pay for baby-sitting.
37. Write a polynomial expression to represent Wei-Ling's total weekly pay.
38. Suppose Wei-Ling makes $5.50 an hour at both jobs. How much was her weekly pay after taxes?

GEOMETRY **For Exercises 39 and 40, use the figure at the right.**



39. Find the sum of the measures of the angles.
40. **MULTI STEP** The sum of the measures of the angles in any quadrilateral is 360°. Find the measure of each angle.

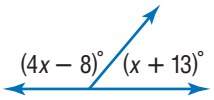
41. **CRITICAL THINKING** If $(3a - 5b) + (2a + 3b) = 5a - 2b$, then what is $(5a - 2b) - (3a - 5b)$? Explain.

Spiral Review with Standardized Test Practice

42. **MULTIPLE CHOICE** What is the sum of $14n + m$ and $n - 9m$?

(A) $13n - 8m$ (B) $14n - 9m$ (C) $15n - 10m$ (D) $15n - 8m$

43. **SHORT RESPONSE** Find the measure of each angle in the figure at the right.



Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*. (Lesson 12-3)

44. $3t + 2s + s + 8t$
45. $7v - 10w + 2$
46. $6f + 9e - 2e + 16$
47. $4q^2 - q - 7 + 6q + 2$

48. **SKYDIVING** The distance d a skydiver falls in t seconds is given by the function $d = 16t^2$. Graph this function and estimate how far a skydiver will fall in 5.5 seconds. (Lesson 12-2)

Find the total amount in each account to the nearest cent. (Lesson 5-8)

49. $250 at 4% for 2.5 years
50. $760 at 5% for 10 months
51. $375 at 9.4% for 14 years
52. $1,200 at 2.2% for $3\frac{1}{3}$ years

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Rewrite each expression as an addition expression by using the additive inverse.** (Lessons 1-4 and 1-5)

53. $6 - 7$
54. $a^2 - 8$
55. $4x - 5y$
56. $(c + d) - 3c$

CHAPTER 12

Mid-Chapter Practice Test

Vocabulary and Concepts

1. **Describe** the difference between the graphs of linear functions and nonlinear functions. (Lesson 12-1)
2. **OPEN ENDED** Write two polynomials whose sum is $5x - 3y$. (Lesson 12-4)

Skills and Applications

Determine whether each equation or table represents a *linear* or *nonlinear* function. Explain. (Lesson 12-1)

3. $3y = x$
4. $y = 5x^3 + 2$
5.

x	1	3	5	7
y	−5	−6	−7	−8

6.

x	−1	0	1	2
y	1	0	1	4

Graph each function. (Lesson 12-2)

7. $y = 2x^2$
8. $y = -x^2 + 3$
9. $y = 4x^2 - 1$

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*. (Lesson 12-3)

10. $3x + 2 - 5x + 1$
11. $6a^2 + 5x - 2a$
12. $y^2 + 3y + 1 + 5y - 2y^2$
13. $3x^2 - 6x + 5x + 8$

Add. (Lesson 12-4)

14. $(3a + 6) + (2a - 5)$
15. $(-3x - 2) + (-2x + 5)$
16. $(3q^2 - 5) + (2q^2 - q)$
17. $(a^2 - 2a + 3) + (3a^2 - 5a + 6)$

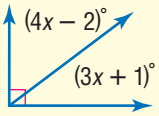
18. **AMUSEMENT PARK RIDES** Your height h above the ground t seconds after being released at the top of a free-fall ride is given by the function $h = -16t^2 + 200$. Graph this function. After about how many seconds will the ride be 60 feet above the ground? (Lesson 12-2)

Standardized Test Practice

19. **MULTIPLE CHOICE** Which expression is *not* a monomial? (Lesson 12-3)

(A) -6 (B) x^3 (C) $4a$ (D) $\frac{4}{n}$

20. **SHORT RESPONSE** Find the measure of each angle in the figure below. (Lesson 12-4)



The Game Zone

A Place To Practice Your Math Skills

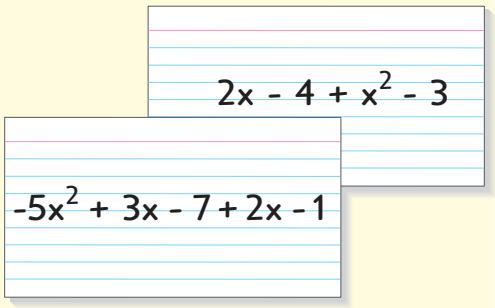
Polynomial Challenge

GET READY!

Players: four
Materials: stopwatch, algebra tiles, scissors, 10 index cards

GET SET!

- Cut each index card in half to make 20 playing cards.
- Each player should write a polynomial with four or five terms that can be modeled with algebra tiles on each of five cards.
- At least two of the polynomials should contain one or more positive or negative x^2-terms.



GO!

- Mix the cards and place the stack facedown on the table.
- The first player turns over the top two cards and lays them side-by-side on the table.
- The player then has one minute to model the two polynomials using algebra tiles and find the sum.
- If the player is correct, he or she scores one point. Those cards are then placed in a discard pile and it becomes the next player's turn.
- **Who Wins?** The first player to score 5 points wins.

John Evans

12-5 Subtracting Polynomials

What You'll LEARN

Subtract polynomials.

REVIEW Vocabulary

additive inverse: a number and its opposite (Lesson 1-4)

HANDS-ON Mini Lab

Materials
- algebra tiles

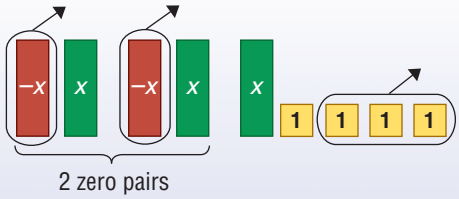
Work with a partner.

You can use algebra tiles to find $(x + 4) - (-2x + 3)$.

STEP 1 Model the polynomial $x + 4$.

STEP 2 To subtract $-2x + 3$, you need to remove 2 negative x-tiles and 3 1-tiles.

STEP 3 Since there are no negative x-tiles to remove, add 2 zero pairs of x-tiles. Then remove 2 negative x-tiles and 3 1-tiles.



1. From the tiles that remain, determine the value of $(x + 4) - (-2x + 3)$.
2. Use algebra tiles to find $(2x^2 + 3x + 5) - (x^2 - x + 2)$.

As with adding polynomials, to subtract two polynomials, you subtract the like terms.

EXAMPLES Subtract Polynomials

Subtract.

1 $(7a + 5) - (3a + 4)$

$$\begin{array}{rl} 7a + 5 & \\ (-)\ 3a + 4 & \text{Align like terms.} \\ \hline 4a + 1 & \text{Subtract.} \end{array}$$

The difference is $4a + 1$.

2 $(5x^2 + 3x + 4) - (3x^2 - 2)$

$$\begin{array}{rl} 5x^2 + 3x + 4 & \\ (-)\ 3x^2 \quad\quad - 2 & \text{Align like terms.} \\ \hline 2x^2 + 3x + 6 & \text{Subtract.} \end{array}$$

The difference is $2x^2 + 3x + 6$.

Recall that you can subtract a number by adding its *additive inverse.* You can also subtract a polynomial by adding its additive inverse. To find the additive inverse of a polynomial, find the opposite of each term.

Polynomial	Terms	Opposites	Additive Inverse
$x + 5$	x, 5	$-x$, -5	$-x - 5$
$-x^2 - 4x + 2$	$-x^2$, $-4x$, 2	x^2, $4x$, -2	$x^2 + 4x - 2$

EXAMPLES Subtract Using the Additive Inverse

3 **Find $(4x + 9) - (7x - 2)$.**

The additive inverse of $7x - 2$ is $-7x + 2$.

$(4x + 9) - (7x - 2)$

$= (4x + 9) + (-7x + 2)$ To subtract $(7x - 2)$, add $(-7x + 2)$.

$= (4x - 7x) + (9 + 2)$ Group like terms.

$= -3x + 11$ Simplify by combining like terms.

The difference is $-3x + 11$.

4 **Find $(6y^2 - 5) - (-3y + 4)$.**

The additive inverse of $-3y + 4$ is $3y - 4$.

$$\begin{array}{rr} 6y^2 & -5 \\ (-) & -3y + 4 \end{array} \rightarrow \begin{array}{rr} 6y^2 & -5 \\ (+) & 3y - 4 \\ \hline & 6y^2 + 3y - 9 \end{array}$$

The difference is $6y^2 + 3y - 9$.

Your Turn **Subtract.**

a. $(5p + 3) - (12p - 8)$

b. $(x^2 - 6x + 4) - (2x^2 - 7x - 1)$

REAL-LIFE CAREERS

How Does an Automotive Engineer Use Math?

Automotive engineers use polynomials to model a car's speed under different road conditions.

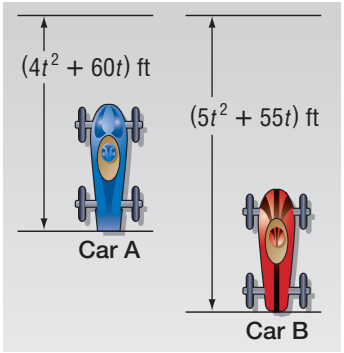
Research

For information about a career as an automotive engineer, visit: msmath3.net/careers

EXAMPLE Use Polynomials to Solve a Problem

5 **CARS** **Car A travels a distance of $4t^2 + 60t$ feet t seconds after the start of a soapbox derby. Car B travels $5t^2 + 55t$ feet. How far apart are the two cars 8 seconds after the start of the race?**

Write an expression for the difference of the distances traveled by each car.



Words	car B's distance minus car A's distance
Variables	t = the time in seconds
Expression	$(5t^2 + 55t) - (4t^2 + 60t)$

$$\begin{array}{rr} 5t^2 & +55t \\ (-) & (4t^2 + 60t) \end{array} \rightarrow \begin{array}{rr} 5t^2 & +55t \\ (+) & (-4t^2 - 60t) \\ \hline & t^2 - 5t \end{array}$$

Now evaluate this expression for a time of 8 seconds.

$t^2 - 5t = (8)^2 - 5(8)$ Replace t with 8.

$= 64 - 40$ or 24 Simplify.

After 8 seconds, the cars are 24 feet apart.

Skill and Concept Check

1. **NUMBER SENSE** Write the opposite of each term in $4x^2 - 8x + 9$. Then write the additive inverse of this polynomial.

2. **OPEN ENDED** Write two polynomials whose difference is $3x - 8$.

3. **FIND THE ERROR** Karen and Yoshi are finding $(3a^2 - 3a + 5) - (2a^2 + a - 1)$. Who is correct? Explain.

Karen

$(3a^2 - 3a + 5) - (2a^2 + a - 1)$
$= (3a^2 - 3a + 5) + (-2a^2 - a + 1)$
$= a^2 - 4a + 6$

Yoshi

$(3a^2 - 3a + 5) - (2a^2 + a - 1)$
$= (3a^2 - 3a + 5) + (-2a^2 + a - 1)$
$= a^2 - 2a + 4$

GUIDED PRACTICE

Subtract.

4. $\begin{array}{r} 5z + 2 \\ (-)\ 3z + 1 \\ \hline \end{array}$

5. $\begin{array}{r} 7c^2 + c + 5 \\ (-)\ 2c^2 \quad + 4 \\ \hline \end{array}$

6. $\begin{array}{r} 2m^2 + 6m + 8 \\ (-)\ m^2 + 3m - 1 \\ \hline \end{array}$

7. $(6p + 2) - (p - 1)$

8. $(x^2 - x - 4) - (-x + 1)$

9. $(5n^2 + n - 2) - (3n^2 + 2n - 1)$

10. $(r^2 + r - 1) - (2r^2 - r + 2)$

11. Find the difference of $-4a + 5$ and $a - 1$.

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
12–17	1, 2
18–25	3, 4
32–40	5

Extra Practice
See pages 646, 659.

Subtract.

12. $\begin{array}{r} 3x + 6 \\ (-)\ 2x + 5 \\ \hline \end{array}$

13. $\begin{array}{r} 9w + 15 \\ (-)\ 4w + 12 \\ \hline \end{array}$

14. $\begin{array}{r} 8g^2 + 8g + 5 \\ (-)\ 7g^2 + 5g + 1 \\ \hline \end{array}$

15. $\begin{array}{r} 10b^2 - 4b + 9 \\ (-)\ 5b^2 + \ b + 3 \\ \hline \end{array}$

16. $\begin{array}{r} 4u^2 + 3u + 2 \\ (-)\ 2u^2 \quad - 4 \\ \hline \end{array}$

17. $\begin{array}{r} 7y^2 + y + 6 \\ (-)\ 5y^2 \quad + 1 \\ \hline \end{array}$

18. $(10h + 4) - (2h - 3)$

19. $(6a + 6) - (-a + 8)$

20. $(4m^2 - 8) - (-3m + 2)$

21. $(5k^2 - 7) - (9k + 13)$

22. $(c^2 - 2c + 1) - (c^2 + c - 5)$

23. $(3r^2 + r - 1) - (r^2 - r + 3)$

24. Find the difference of $7x^2 + 12x - 9$ and $4x^2 - 3$.

25. What is $-z - 5$ subtracted from $16z - 7$?

Subtract. Then evaluate the difference if $x = -8$ and $y = 5$.

26. $(4x + 10) - (3x + 7)$

27. $(6y - 2) - (2y + 6)$

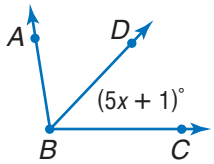
28. $(-3x - 8) - (y - 5)$

29. $(9x + 2y) - (8x - 4)$

30. $(x + 5y) - (-4x + 3y)$

31. $(-2x - y) - (-6x - 3y)$

32. **GEOMETRY** The measure of $\angle ABC$ is $(12x - 8)°$. Write an expression in simplest form for the measure of $\angle ABD$.



FAST FOOD **For Exercises 33–36, use the following information.** Khadijah ordered 3 burritos and 7 tacos from a fast-food drive through. When Khadijah looked at her receipt, she discovered that she had been charged for 5 burritos and 5 tacos.

33. If burritos cost b dollars and tacos cost t dollars, write an expression for the amount Khadijah was charged.
34. Write an expression for the cost of the food she ordered.
35. Write an expression for the amount Khadijah was overcharged.
36. If tacos cost \$0.99 and burritos cost \$1.59, how much was she overcharged?

FUND-RAISER **For Exercises 37–40, use the following information.** Your club spends \$200 on a pizza fund-raiser kit. Each pizza costs you \$6.50 to make. You sell each pizza for \$10.

37. Write a polynomial that models your total expenses for making x pizzas.
38. Write a polynomial that models your income from selling x pizzas.
39. Write a polynomial that models your profit from selling x pizzas. (*Hint*: Profit = Income − Expenses)
40. How much profit will you make if you sell 150 pizzas?

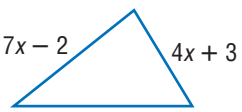
41. **CRITICAL THINKING** Suppose A and B represent polynomials. If $A + B = 7x + 4$ and $A - B = 3x + 2$, find A and B.

Spiral Review with Standardized Test Practice

42. **MULTIPLE CHOICE** Write the additive inverse of $n^2 - 2n + 3$.

(A) $n^2 + 2n - 3$ (B) $-n^2 + 2n - 3$
(C) $-n^2 - 2n - 3$ (D) $n^2 + 2n + 3$

43. **SHORT RESPONSE** The perimeter of the triangle is $16x - 7$ units. Write an expression for the missing length.



Add. (Lesson 12-4)

44. $(7b + 2) + (-5b + 3)$ 45. $(6v^2 - 4) + (v - 1)$ 46. $(t^2 - 8t) + (t^2 + 5)$

SCHOOL **For Exercises 47 and 48, use the following information.** The drama club is selling flowers. The sales for the first two weeks are shown in the table. (Lesson 12-3)

Number of Flowers Sold		
Week	Carnations	Roses
1	54	38
2	65	42

47. The selling prices of a carnation and a rose are C and R respectively. Write a polynomial expression for the total sales.
48. If carnations cost \$2 each and roses cost \$5 each, what was the total amount of sales?

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Write each expression using exponents.** (Lesson 2-8)

49. $3 \cdot 3 \cdot 3 \cdot 3$ 50. $5 \cdot 4 \cdot 5 \cdot 5 \cdot 4$ 51. $7 \cdot (7 \cdot 7)$ 52. $(2 \cdot 2) \cdot (2 \cdot 2 \cdot 2)$

12-6 Multiplying and Dividing Monomials

What You'll LEARN

Multiply and divide monomials.

MATH Symbols

x^5 ← exponent
↑ — base

x^5 x to the fifth power

WHEN am I ever going to use this?

SCIENCE The pH of a solution describes its acidity. Neutral water has a pH of 7. Lemon juice has a pH of 2. Each one-unit decrease in the pH means that the solution is 10 times more acidic. So a pH of 8 is 10 times more acidic than a pH of 9.

pH	Times More Acidic Than a pH of 9	Written Using Powers
8	10	10^1
7	$10 \times 10 = 100$	$10^1 \times 10^1 = 10^2$
6	$10 \times 10 \times 10 = 1{,}000$	$10^1 \times 10^2 = 10^3$
5	$10 \times 10 \times 10 \times 10 = 10{,}000$	$10^1 \times 10^3 = 10^4$
4	$10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$	$10^1 \times 10^4 = 10^5$

1. Examine the exponents of the factors and the exponents of the products in the last column. What do you observe?

Exponents are used to show repeated multiplication. You can use this fact to help find a rule for multiplying powers with the same base.

$$3^2 \cdot 3^4 = \underbrace{(3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3)}_{\text{6 factors}} \text{ or } 3^6$$

(2 factors, 4 factors)

Notice the sum of the original exponents and the exponent in the final product. This relationship is stated in the following rule.

Noteables™ **Key Concept: Product of Powers**

Words To multiply powers with the same base, add their exponents.

Symbols

Arithmetic	Algebra
$2^4 \cdot 2^3 = 2^{4+3}$ or 2^7	$a^m \cdot a^n = a^{m+n}$

STUDY TIP

Common Error
When multiplying powers, do not multiply the bases. $4^5 \cdot 4^2 = 4^7$, not 16^7.

EXAMPLE 1 Multiply Powers

Find $5^2 \cdot 5$. Express using exponents.

$5^2 \cdot 5 = 5^2 \cdot 5^1$ $5 = 5^1$

$= 5^{2+1}$ The common base is 5.

$= 5^3$ Add the exponents.

Check $5^2 \cdot 5 = (5 \cdot 5) \cdot 5$

$= 5 \cdot 5 \cdot 5$

$= 5^3$ ✔

CONTENTS

EXAMPLE 2 Multiply Monomials

Find $-3x^2(4x^5)$. Express using exponents.

$-3x^2(4x^5) = (-3 \cdot 4)(x^2 \cdot x^5)$ Commutative and Associative Properties

$= (-12)(x^{2+5})$ The common base is x.

$= -12x^7$ Add the exponents.

Your Turn **Multiply. Express using exponents.**

a. $9^3 \cdot 9^2$ b. $y^4 \cdot y^9$ c. $-2m(-8m^5)$

There is also a rule for dividing powers that have the same base.

Noteables™ **Key Concept: Quotient of Powers**

Words To divide powers with the same base, subtract their exponents.

Symbols

Arithmetic	Algebra
$\frac{3^7}{3^3} = 3^{7-3}$ or 3^4	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

EXAMPLES 3 4 Divide Powers

Divide. Express using exponents.

3 $\frac{4^8}{4^2}$

$\frac{4^8}{4^2} = 4^{8-2}$ The common base is 4.

$= 4^6$ Simplify.

4 $\frac{n^9}{n^4}$

$\frac{n^9}{n^4} = n^{9-4}$ The common base is n.

$= n^5$ Simplify.

Your Turn **Divide. Express using exponents.**

d. $\frac{5^7}{5^4}$ e. $\frac{x^{10}}{x^3}$ f. $\frac{12w^5}{2w}$

REAL-LIFE MATH

SOUND The decibel measure of the loudness of a sound is the exponent of its relative intensity multiplied by 10. A jet engine has a loudness of 120 decibels.

EXAMPLE 5 Divide Powers to Solve a Problem

5 **SOUND The loudness of a conversation is 10^6 times as intense as the loudness of a pin dropping, while the loudness of a jet engine is 10^{12} times as intense. How many times more intense is the loudness of a jet engine than the loudness of a conversation?**

To find how many times more intense, divide 10^{12} by 10^6.

$\frac{10^{12}}{10^6} = 10^{12-6}$ Quotient of Powers

$= 10^6$ Simplify.

The loudness of a jet engine is 10^6 or 1,000,000 times as intense as the loudness of a conversation.

Skill and Concept Check

1. **Writing Math** Determine whether the following statement is *true* or *false*.
If you change the order in which you multiply two monomials, the product will be different.
Explain your reasoning or give a counterexample.

2. **OPEN ENDED** Write a multiplication expression whose product is 4^{15} and a division expression whose quotient is 4^{15}.

3. **NUMBER SENSE** Is $\frac{2^{100}}{2^{99}}$ greater than, less than, or equal to 2?

GUIDED PRACTICE

Multiply or divide. Express using exponents.

4. $4^5 \cdot 4^3$
5. $3^6 \cdot 3$
6. $n^2 \cdot n^9$
7. $-2a(3a^4)$
8. $\frac{7^6}{7}$
9. $\frac{9c^7}{3c^2}$

Practice and Applications

Multiply or divide. Express using exponents.

10. $6^8 \cdot 6^5$
11. $7^3 \cdot 7^3$
12. $2^9 \cdot 2$
13. $11 \cdot 11^4$
14. $n \cdot n^7$
15. $b^{13} \cdot b$
16. $2g \cdot 7g^6$
17. $(3x^8)(5x)$
18. $-4a^5(6a^5)$
19. $(8w^4)(-w^7)$
20. $(-p)(-9p^2)$
21. $-5y^3(-8y^6)$
22. $\frac{3^9}{3^2}$
23. $\frac{4^{10}}{4^5}$
24. $\frac{8^4}{8}$
25. $\frac{10^{12}}{10}$
26. $\frac{r^7}{r^2}$
27. $\frac{x^{14}}{x^8}$
28. $\frac{14n^6}{7n}$
29. $\frac{24k^3}{8k^2}$
30. $xy^2(x^3y)$
31. $4a^2b^3(7ab^2)$
32. $\frac{20a^5b}{4ab}$
33. $\frac{16x^3y^2}{2x^2y}$

HOMEWORK HELP

For Exercises	See Examples
10–13, 34–35	1
14–21	2
22–33, 36–37	3, 4
38–43	5

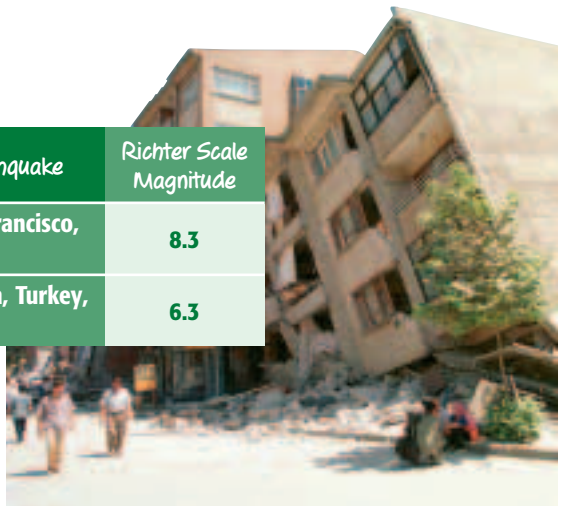
Extra Practice
See pages 647, 659.

34. the product of seven to the tenth power and seven cubed
35. the quotient of n to the sixth power and n squared
36. What is the product of 2^6, 2, and 2^3?
37. Find $x^3 \cdot x^9 \div x^5$.

EARTHQUAKES For Exercises 38 and 39, use the information in the table at the right and below.
For each increase on the Richter scale, an earthquake's vibrations, or *seismic waves*, are 10 times greater.

Earthquake	Richter Scale Magnitude
San Francisco, 1906	8.3
Adana, Turkey, 1998	6.3

38. How many times greater are the seismic waves of an earthquake with a magnitude of 6 than an aftershock with a magnitude of 3?
39. How many times greater were the seismic waves of the 1906 San Francisco earthquake than the 1998 Adana, Turkey, earthquake?

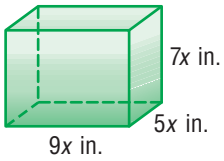


CONTENTS

40. **LIFE SCIENCE** A cell culture contains 2^6 cells. By the end of the day, there are 2^{10} times as many cells in the culture. How many cells are there in the culture by the end of the day?

41. **ASTRONOMY** Venus is approximately 10^8 kilometers from the Sun. The gas giant Saturn is more than 10^9 kilometers from the Sun. About how many times farther away from the Sun is Saturn than Venus?

42. **GEOMETRY** Find the volume of the rectangular prism.



43. **POPULATION** The continent of North America contains approximately 10^7 square miles of land. If the population doubles, there will be about 10^9 people on the continent. At that point, how many people will be on each square mile of land?

44. **CRITICAL THINKING** What is half of 2^{30}? Write your answer using exponents.

CRITICAL THINKING **Divide.**

45. $\frac{a^8}{a^8}$ 46. $\frac{n^2}{n^5}$ 47. $\frac{6x^7y^4}{3x^3y^9}$ 48. $\frac{y^6}{y^{-4}}$ 49. $\frac{a^8b^{-1}c^{-5}}{a^3b^2c^{-3}}$

Spiral Review with Standardized Test Practice

50. **MULTIPLE CHOICE** Find the product of $-5x^2$ and $-6x^8$.

(A) $-11x^{10}$ (B) $-30x^{16}$ (C) $11x^{10}$ (D) $30x^{10}$

51. **MULTIPLE CHOICE** Find $\frac{(-2)^5}{(-2)^4}$.

(F) $(-2)^9$ (G) -2 (H) 1 (I) 2^9

Subtract. (Lesson 12-5)

52. $(3x + 8) - (5x + 1)$ 53. $(5a - 2) - (3a - 4)$ 54. $(6y^2 + 3y + 9) - (2y^2 + 8y + 1)$

SCHOOL **For Exercises 55 and 56, use the following information and the table at the right.**
Suppose your total number of grade points for the first semester was $2A + 2B + C$ and your total for the second semester was $A + 3B + D$. (Lesson 12-4)

Grade	Grade Points
A	4
B	3
C	2
D	1
F	0

55. Add the polynomials to find your total grade points for the year.

56. Evaluate the sum by substituting the grade point value for each variable.

Find the mean, median, and mode of each set of data. Round to the nearest tenth if necessary. (Lesson 9-4)

57. 52, 57, 52, 33, 39, 43, 53 58. 19, 28, 25, 64, 64, 76, 18

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Use the Distributive Property to write each expression as an equivalent algebraic expression.** (Lesson 10-1)

59. $3(x + 4)$ 60. $5(y - 2)$ 61. $-2(n + 8)$ 62. $-4(p - 6)$

12-7a Problem-Solving Strategy

A Preview of Lesson 12-7

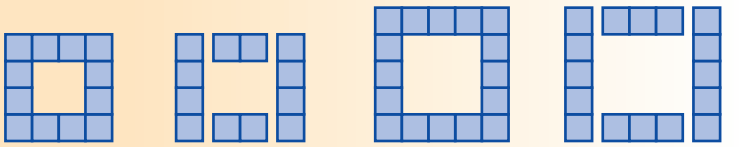
Make a Model

What You'll LEARN

Solve problems by making a model.

We need to arrange some of these square tables into a square that is open in the middle and has 10 tables on each side.

We have 35 tables. Do we have enough? Let's **make a model** using these tiles.

Explore	We want to know how many square tables it will take to make the outline of a 10-by-10 square.
Plan	Let's start by making a model of a 4-by-4 square and of a 5-by-5 square. Then, let's look for a pattern.
Solve	 4-by-4 square; 2 groups of 4 and 2 groups of 2; 5-by-5 square; 2 groups of 5 and 2 groups of 3 For a 10-by-10 square we need $2 \times 10 + 2 \times 8$ or 36. We have 35 tables, so we need one more.
Examine	We get the same answer when we make 4 groups of tiles. Each group has 1 less tile than the length of the square. Since $4(10 - 1)$ is 36, our answer is reasonable.  4 groups of 3

Analyze the Strategy

1. **Explain** why building a model is an appropriate strategy for solving the problem.
2. **Draw a diagram** showing another way the students could have grouped the tiles to solve this problem. Use a 4 by 4 square.
3. **Write** a problem that can be solved by making a model. Describe the model. Then solve the problem.

CONTENTS

Apply the Strategy

Solve. Make a model.

4. **STICKERS** In how many different ways can three rectangular stickers be torn from a sheet of such stickers so that all three stickers are attached to each other? Draw each arrangement.

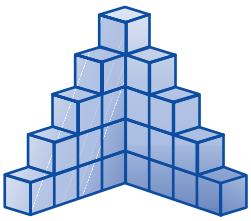
5. **GEOMETRY** A 10-inch by 12-inch piece of cardboard has a 2-inch square cut out of each corner. Then the sides are folded up and taped together to make an open box. Find the volume of the box.

Mixed Problem Solving

Solve. Use any strategy.

6. **PETS** Mrs. Harper owns both cats and canaries. Altogether her pets have thirty heads and eighty legs. How many cats does she have?

TOWER For Exercises 7 and 8, use the figure at the right.



7. How many cubes would it take to build this tower?

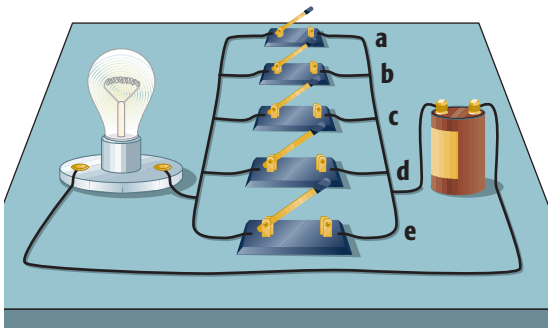
8. How many cubes would it take to build a similar tower that is 12 cubes high?

9. **CARS** Yesterday you noted that the mileage on the family car read 60,094.8 miles. Today it reads 60,099.1 miles. Was the car driven about 4 or 40 miles?

10. **HOBBIES** Lorena says to Angela, "If you give me one of your baseball cards, I will have twice as many baseball cards as you have." Angela answers, "If you give me one of your cards, we will have the same numbers of cards." How many cards do each of the girls have?

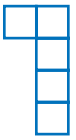
11. **PARKING** Campus parking space numbers consist of three digits. They are typed on a slip of paper and given to students at orientation. Tara accidentally read her number upside-down. The number she read was 723 more than her actual parking space number. What is Tara's parking space number?

12. **SCIENCE** The light in the circuit will turn on if one or more switches are closed. How many combinations of open and closed switches will result in the light being on?



13. **CAMP** The camp counselor lists 21 chores on separate pieces of paper and places them in a basket. The counselor takes one piece of paper, and each camper takes one as the basket is passed around the circle. There is one piece of paper left when the basket returns to the counselor. How many people could be in the circle if the basket goes around the circle more than once?

14. **STANDARDIZED TEST PRACTICE**



In how many different ways can five squares be arranged to form a single shape so that touching squares border on a full side? One arrangement is shown at the right.

(A) 8 (B) 12

(C) 16 (D) 20

12-7 Multiplying Monomials and Polynomials

What You'll LEARN

Multiply monomials and polynomials.

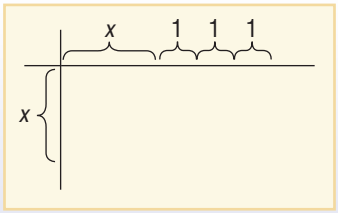
HANDS-ON **Mini Lab**

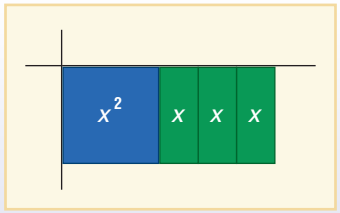
Materials
- algebra tiles
- product mat

Algebra tiles can be used to form a rectangle whose length and width each represent a polynomial. The area of the rectangle is the product of the polynomials. Use algebra tiles to find $x(x + 3)$.

STEP 1 Use algebra tiles to mark off a rectangle with a width of x and a length of $x + 3$ on a product mat.

STEP 2 Using the marks as a guide, fill in the rectangles with algebra tiles.





1. What is $x(x + 3)$ in simplest form?

Use algebra tiles to find each product.

2. $x(x + 4)$ 3. $x(3x + 1)$ 4. $2x(x + 3)$

In Lesson 10-1, you learned how to rewrite an expression like $4(x + 3)$ using the Distributive Property. This is an example of multiplying a polynomial by a monomial.

polynomial

monomial → $4(x + 3) = 4(x) + 4(3)$ Distributive Property

$= 4x + 12$ Simplify.

Often, the Distributive Property and the definition of exponents are needed to simplify the product of a monomial and a polynomial.

EXAMPLE 1 Use the Distributive Property

Find $x(x + 2)$.

$x(x + 2) = x(x) + x(2)$ Distributive Property

$= x^2 + 2x$ $x \cdot x = x^2$

EXAMPLE Use the Distributive Property

2 **Find $-5y(y + 8)$.**

$-5y(y + 8) = -5y(y) + (-5y)(8)$ Distributive Property

$= -5y^2 + (-40y)$ $-5 \cdot y \cdot y = -5y^2$

$= -5y^2 - 40y$ Definition of subtraction

Your Turn **Multiply.**

a. $n(n - 9)$ **b.** $(10 + 2p)4p$ **c.** $-3x(6x - 4)$

Sometimes you may need to use the Product of Powers rule.

EXAMPLES Use the Product of Powers Rule

3 **Find $3n(n^2 - 7)$.**

$3n(n^2 - 7) = 3n[n^2 + (-7)]$ Rewrite $n^2 - 7$ as $n^2 + (-7)$.

$= 3n(n^2) + 3n(-7)$ Distributive Property

$= 3n^3 + (-21n)$ $3n(n^2) = 3n^{1+2}$ or $3n^3$

$= 3n^3 - 21n$ Definition of subtraction

4 **Find $2x(x^2 + 3x - 5)$.**

$2x(x^2 + 3x - 5)$

$= 2x[x^2 + 3x + (-5)]$ Rewrite $x^2 + 3x - 5$ as $x^2 + 3x + (-5)$.

$= 2x(x^2) + 2x(3x) + 2x(-5)$ Distributive Property

$= 2x^3 + 6x^2 + (-10x)$ Simplify.

$= 2x^3 + 6x^2 - 10x$ Definition of subtraction

Your Turn **Multiply.**

d. $5y(4y^2 - 2y)$ **e.** $a(a^2 - 4a + 6)$ **f.** $-4p(2p^2 - p + 3)$

Skill and Concept Check

1. **OPEN ENDED** Write a polynomial with three terms and a monomial that contains a variable with a power of 1. Then find their product.

2. **FIND THE ERROR** Christopher and Stephanie are finding the product of $3x$ and $2x^2 - 3x + 8$. Who is correct? Explain.

Christopher
$3x(2x^2 - 3x + 8)$
$= 6x^2 - 9x + 24$

Stephanie
$3x(2x^2 - 3x + 8)$
$= 6x^3 - 9x^2 + 24x$

GUIDED PRACTICE

Multiply.

3. $m(m + 5)$
4. $(2w - 1)(3w)$
5. $-4x(x + 1)$
6. $k(k^2 - 7)$
7. $g(2g^2 - 5g + 9)$
8. $3z(4z^2 - 6z - 10)$

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
9–18, 25–26	1, 2
19–24	3, 4

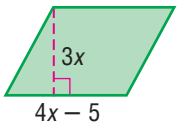
Extra Practice
See pages 647, 659.

Multiply.

9. $r(r + 9)$
10. $t(t - 4)$
11. $(3b - 2)(3b)$
12. $(5x + 1)(2x)$
13. $-6d(d + 5)$
14. $-a(7a - 8)$
15. $6h(4 + 3h)$
16. $8w(1 - 7w)$
17. $11e(2e - 7)$
18. $10a(5a + 5)$
19. $4y(y^2 - 9)$
20. $-6g(2g^2 + 1)$
21. $t(t^2 + 5t + 9)$
22. $-n(3n^2 - 4n + 13)$
23. $-2r(4r^2 - r - 8)$
24. $11c(6c^2 - 8c + 1)$

25. **GARDENING** A square garden plot measures x feet on each side. Suppose you double the length of the plot and increase the width by 4 feet. Write two expressions for the area of the new plot.

26. **GEOMETRY** Write an expression in simplest form for the area of the figure.



27. **CRITICAL THINKING** Draw a model showing how algebra tiles can be used to find the following product of two *binomials*, or polynomials with two terms: $(x + 2)(x + 3)$.

Spiral Review with Standardized Test Practice

28. **MULTIPLE CHOICE** What is the product of $4x^2$ and $x^2 + 2x - 3$?

(A) $4x^2 + 8x - 12$
(B) $4x^4 + 8x^2 - 12x$
(C) $4x^4 + 8x^3 - 12x^2$
(D) $5x^2 + 6x + 1$

29. **SHORT RESPONSE** The length of a rectangle is twice its width. If the width is x, write an equation for the area A of the rectangle. Then graph the area as a function of x.

Multiply or divide. Express using exponents. (Lesson 12-6)

30. $5^2 \cdot 5$
31. $\frac{11^8}{11^5}$
32. $3x^3 \cdot 9x^3$
33. $\frac{21a^5}{3a^4}$

34. **BUSINESS** Allison's income from selling x beaded bracelets is $6.50x$. Her expenses are $4x + 35$. Write an expression for her profit. (Lesson 12-5)

INTERDISCIPLINARY PROJECT

Getting Down to Business

Math and Economics It's time to complete your project. Use the information and data you have gathered about the cost of materials and the feedback from your peers to prepare a video or brochure. Be sure to include a scatter plot with your project.

WebQuest msmath3.net/webquest

CHAPTER 12

Study Guide and Review

Vocabulary and Concept Check

monomial (p. 570) nonlinear function (p. 560) polynomial (p. 570) quadratic function (p. 565)

State whether each sentence is *true* or *false*. If *false*, replace the underlined word or number to make a true sentence.

1. The expression $x^2 - 3x$ is an example of a monomial.
2. A nonlinear function has a constant rate of change.
3. To multiply two polynomials, you combine like terms.
4. A quadratic function is a nonlinear function.
5. To divide powers with the same base, subtract the exponents.

Lesson-by-Lesson Exercises and Examples

12-1 Linear and Nonlinear Functions (pp. 560–563)

Determine whether each equation or table represents a *linear* or *nonlinear* function. Explain.

6. $y - 4x = 1$
7. $y = x^2 + 3$
8.

x	2	3	4	5
y	1	3	7	12

Example 1 **Determine whether the table represents a *linear* or *nonlinear* function.**

x	y
−2	−3
−1	−1
0	1
1	3

As x increases by 1, y increases by 2. The rate of change is constant, so this function is linear.

12-2 Graphing Quadratic Functions (pp. 565–568)

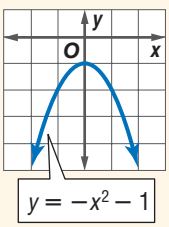
Graph each function.

9. $y = -4x^2$
10. $y = x^2 + 4$
11. **SCIENCE** A ball is dropped from the top a 36-foot tall building. The quadratic equation $d = -16t^2 + 36$ models the distance d in feet the ball is from the ground at time t. Graph the function. Then use your graph to find how long it takes for the ball to reach the ground.

Example 2 **Graph $y = -x^2 - 1$.**

Make a table of values. Then plot and connect the ordered pairs with a smooth curve.

x	$y = -x^2 - 1$	(x, y)
−2	$-(-2)^2 - 1$	(−2, −5)
−1	$-(-1)^2 - 1$	(−1, −2)
0	$-(0)^2 - 1$	(0, −1)
1	$-(1)^2 - 1$	(1, −2)
2	$-(2)^2 - 1$	(2, −5)



Mixed Problem Solving
For mixed problem-solving practice, see page 659.

12-3 Simplifying Polynomials (pp. 570–573)

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*.

12. $3a - b - 7a + 2 + 4b$
13. $8x - y + 1$
14. $3n^2 + 7n - 4n^2 + n$

Example 3 **Simplify $8a^2 - 5a + 6 - 9a^2 - 6$.**

$$\begin{aligned} &8a^2 - 5a + 6 - 9a^2 - 6 \\ &= 8a^2 + (-5a) + 6 + (-9a^2) + (-6) \\ &= [8a^2 + (-9a^2)] + (-5a) + [6 + (-6)] \\ &= -1a^2 + (-5a) + 0 \text{ or } -a^2 - 5a \end{aligned}$$

12-4 Adding Polynomials (pp. 574–577)

Add.

15. $(3a^2 + 6a) + (2a^2 - 5a)$
16. $(b^2 - 2b + 4) + (2b^2 + b - 8)$
17. $(10m^2 + 5m - 9) + (-2m + 3)$

Example 4 **Find $(3x^2 - 2) + (2x^2 + 5)$.**

$$\begin{aligned} &(3x^2 - 2) + (2x^2 + 5) \\ &= (3x^2 + 2x^2) + (-2 + 5) && \text{Group like terms.} \\ &= 5x^2 + 3 && \text{Simplify.} \end{aligned}$$

12-5 Subtracting Polynomials (pp. 580–583)

Subtract.

18. $(7g + 2) - (5g + 1)$
19. $(3c - 7) - (-3c + 4)$
20. $(7p^2 + 2p - 5) - (4p^2 + 6p - 2)$
21. $(6k^2 - 3) - (k^2 - 5k - 2)$

Example 5 **Find $(5x - 1) - (6x + 4)$.**

To subtract $6x + 4$, add $-6x - 4$.

$$\begin{aligned} &(5x - 1) - (6x + 4) \\ &= (5x - 1) + (-6x - 4) \\ &= [5x + (-6x)] + [-1 + (-4)] \\ &= -1x + (-5) \\ &= -x - 5 \end{aligned}$$

12-6 Multiplying and Dividing Monomials (pp. 584–587)

Multiply or divide. Express using exponents.

22. $4 \cdot 4^5$
23. $-9y^2(-4y^9)$
24. $\frac{n^5}{n}$
25. $\frac{21c^{11}}{-7c^8}$

Example 6

Find $3a^3 \cdot 4a^7$.

$$\begin{aligned} 3a^2 \cdot 4a^7 &= (3 \cdot 4)a^{3+7} \\ &= 12a^{10} \end{aligned}$$

Example 7

Find $\frac{6^8}{6^3}$.

$$\begin{aligned} \frac{6^8}{6^3} &= 6^{8-3} \\ &= 6^5 \end{aligned}$$

12-7 Multiplying Monomials and Polynomials (pp. 590–592)

Multiply.

26. $a(a - 7)$
27. $(3y + 4)(3y)$
28. $-4n(n - 2)$
29. $p(p^2 - 6)$
30. $x(2x^2 + x - 5)$
31. $-2k(5k^2 - 3k + 8)$

Example 8 **Find $-2x(5x + 3)$.**

$$\begin{aligned} -2x(5x + 3) &= -2x(5x) + (-2x)(3) \\ &= -10x^2 + (-6x) \\ &= -10x^2 - 6x \end{aligned}$$

CHAPTER 12

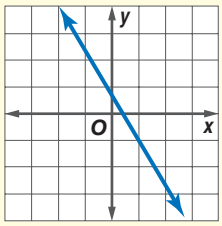
Practice Test

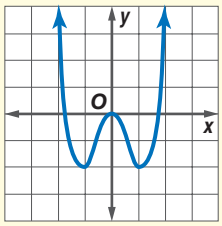
Vocabulary and Concepts

1. **OPEN ENDED** Write two polynomials whose difference is $-4n + 5$.
2. **State** whether the Quotient of Powers rule applies to $\frac{6^5}{3^2}$. Explain.
3. **Describe** the function $y = 3x^2$ using two different terms.

Skills and Applications

Determine whether each graph, equation, or table represents a *linear* or *nonlinear* function. Explain.

4. 

5. 

6. $2x = y$

7. $y = \frac{x}{7} + 3$

8.

x	-3	-1	1	3
y	2	10	18	26

9. Graph the function $y = -2x^2 + 3$.

Simplify each polynomial. If the polynomial cannot be simplified, write *simplest form*.

10. $-6x + 4y - 8 + y - 1$

11. $2a^2 + 4a + 3a^2 + 5a$

12. $10p + 7p^2 + 1$

Add or subtract.

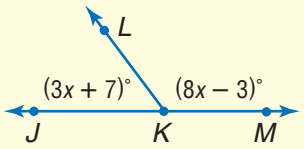
13. $(4c^2 + 2) + (-4c^2 + 1)$

14. $(-x^2 + 2x - 5) + (4x^2 - 6x)$

15. $(9z^2 - 3z) - (5z^2 + 8z)$

16. $(5n^2 - 4n + 1) - (4n - 5)$

17. **GEOMETRY** Write an expression for the measure of $\angle JKM$. Then find the value of x.



Multiply or divide. Express using exponents.

18. $15^3 \cdot 15^5$

19. $-5m^6(-9m^8)$

20. $\frac{3^{15}}{3^7}$

21. $\frac{-40w^8}{8w}$

Multiply.

22. $8n(n + 3)$

23. $g^3(6g - 5)$

24. $-4x(3x^2 - 6x + 8)$

Standardized Test Practice

25. **MULTIPLE CHOICE** Find the value of ● in the equation $\frac{x^9}{x^{\bullet}} = x^3$.

(A) 3 (B) 6 (C) 12 (D) 27

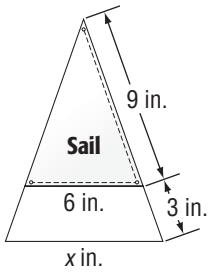
CHAPTER 12

Standardized Test Practice

PART 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Nolan wants to make a larger sail for his model boat. How long will the base of the new sail be in inches? (Lesson 4-5)



(A) 2 (B) 8
(C) 27 (D) 36

2. Which statement is false? (Lesson 5-5)

(F) 42% of 60 is greater than 24.
(G) 31% of 90 is greater than 30.
(H) 79% of 250 is less than 200.
(I) 3% of 80 is less than 3.

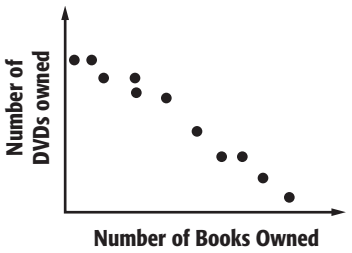
3. A group of dancers form a circle for a routine they are performing. The radius of their circle is 8 yards. If they increase the area of their circle by 4 times, what will be the radius, in yards, of the new circle? (Lesson 7-2)

(A) 2 (B) 12 (C) 16 (D) 32

4. Fourteen dogs are enrolled in dog-training class. All the dogs weigh about 50 pounds, except for Spot, who weighs 35 pounds. How does Spot's weight affect the mean and median weights of the entire class? (Lesson 9-4)

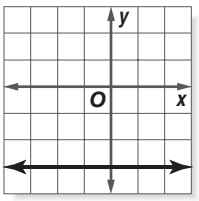
(F) Spot's weight affects the mean more.
(G) Spot's weight affects the median more.
(H) Spot's weight has an equal affect on the mean and median.
(I) Spot's weight has no affect on the mean or median.

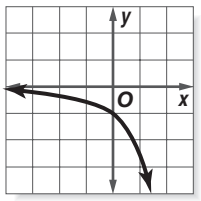
5. On the basis of the graph below, what relationship exists between the number of DVDs a person owns and the number of books they own? (Lesson 11-6)

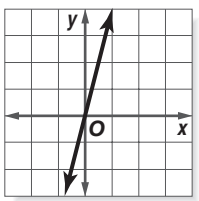


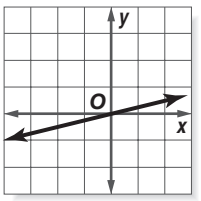
(A) As the number of DVDs owned increases, the number of books owned increases.
(B) As the number of DVDs owned increases, the number of books owned decreases.
(C) There is no relationship between the number of DVDs owned and the number of books owned.
(D) For every DVD owned, there are 3 books owned.

6. Which is the graph of a nonlinear function? (Lesson 12-1)

(F)


(G)


(H)


(I)


7. If $a = b^4$, which expression is equal to b^8? (Lesson 12-6)

(A) a^2 (B) a^4 (C) a^{12} (D) a^{32}

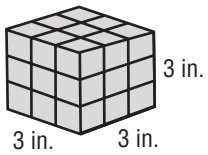
PART 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. A square tile measures 9 inches by 9 inches. What is the least number of tiles needed to cover a rectangular floor measuring 21 feet by 27 feet? (Lesson 1-1)

9. There are 240 students in attendance at a student government conference held in Atlanta. Half of these students are from Georgia. Of the remaining students, $\frac{1}{5}$ are from Alabama, and $\frac{1}{4}$ are from Florida. All others are from Tennessee. How many students are from Tennessee? (Lesson 2-3)

10. The block shown below weighs 54 grams. What would be the weight of a block of the same material that measures 6 inches by 6 inches by 6 inches? (Lesson 7-5)



11. Brad's Internet password is a permutation of his initials, B, W, and D, and the numbers 5, 8, and 2. How many different passwords does he have to choose from if no letter or number is used more than once? (Lesson 8-3)

12. What are the coordinates of the point where the graph of $9 + y = 4x$ intercepts the y-axis? (Lesson 11-5)

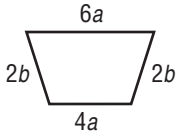
TEST-TAKING TIP

Question 12 Before graphing an equation, determine whether it is necessary to do so in order to answer the question. In Question 12, you can write $9 + y = 4x$ in slope-intercept form and identify its y-intercept to determine where its graph will cross the y-axis.

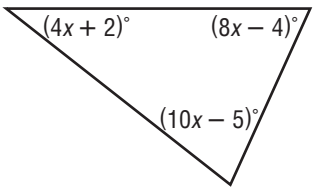
13. Copy and complete the table below so that it represents a linear function. (Lesson 12-1)

x	7	5			
y	−4	1			

14. Write an expression in simplest form for the perimeter of the figure at the right. (Lesson 12-3)



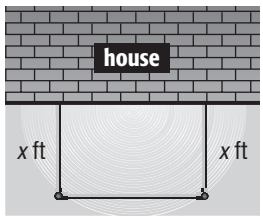
15. Find the measure of the value of x in the figure below. (Lesson 12-4)



PART 3 Extended Response

Record your answers on a sheet of paper. Show your work.

You have 40 feet of fencing to make a rectangular kennel for your dog. You will use your house as one side. (Lessons 12-2 and 12-7)



16. Write an algebraic expression for the kennel's length.

17. Write an algebraic expression in simplest form for the area of the kennel.

18. Write the area A of the kennel as a function of its width x.

19. Make a table of values and graph the function you wrote in Exercise 18.

20. Use your graph to determine the width that produces a kennel with the greatest area.